VALUATION OF PROJECTS AND ENTERPRISES USING DISCOUNTING TECHNIQUES

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1. Introduction

This manual is addressed to all those who, for educational or professional purposes, need to valuate projects or enterprises using the discounted cash flow (DCF) method.

This paper refers to notions and principles broadly used in the academic literature and aims to give practical advice to users; above all, it offers a calculation method that consistently reconciles all of the methods developed by the various currents of thought.

This manual contains various examples that illustrate in detail, using spreadsheets, how to make the calculations and how to verify that the result is the same and consistent with all currently existing methodologies.

The main take-home message for readers is that a project or an enterprise analysed with discounting methodologies must at all times have the same value irrespectively of whether the value has been analysed using the WACC methodology or the EVA methodology or by directly measuring the net cash flows to shareholders. This arises out of the fact that all of these methodologies rest upon the same underlying mathematical grounds and, as a result, refer to each other.

2. The Notion of Risk

Assume, by way of a hypothesis, that you go to a Casino where there are two roulette tables which, as opposed to real ones, statistically allow to win:

- **On table A there is a 10-number wheel which pays the winner 11 times his/her stake.**

- **on table B there is a 100-number wheel which pays the winner 110 times his/her stake.**

So, on average, the win is the same for both tables.

On table A, if you play infinitely, you lose €1 nine times out of ten and win €11 one time out of ten. In practice, 10 bets for a €10 investment would, on average, earn the winner €1, i.e. 10% of his/her investment:

-10 + 9·0 + 1·11 = €1 gain

On average, for every €1 bet, the player gets €1.1, gaining €0.10.

Also on table B, with the 100-number wheel, if you play infinitely, the average win per bet will be 10% of the investment:

-100 + 99·0 + 1·110 = €10 gain

Like on the previous table, €100 invested on table B, in €1 bets, generate, on average, wins equal to 10% of the invested capital, i.e. €0.10 per bet.

Hence, both tables yield the same gain for players. However, these tables are not the same. One realizes this by analyzing what happens with a finite number of bets.

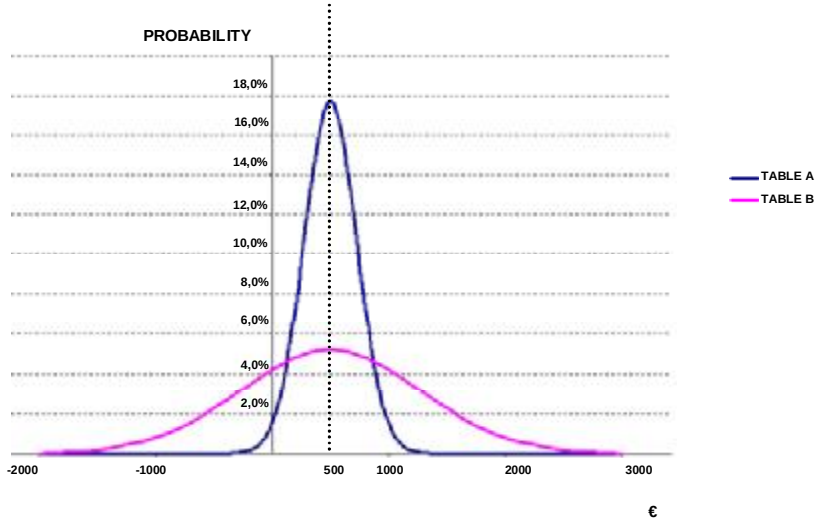
If we had €5,000 to spend on either table, from a purely statistical point of view, we would expect by the end of the night a €500 gain, arising from the 5,000 €1 bets with an average gain of €0.10 each.

If, however, we analyse the normal distribution of the probabilities of winning on the two tables, relating to a series of 5,000 €1 bets, table A with the 10-number wheel has a much tighter distribution of probabilities compared to table B with the 100-number wheel.

This happens for the simple reason that on table B wins are a rarer occurrence than on table A. Therefore, given an infinite number of bets, wins on table B are more uncertain. "Uncertain" does not mean that wins on table B are fewer compared to table A: indeed, they could be much more numerous, as shown by the width and symmetry of the Gaussian curve.

Figure 1 shows the distribution of probabilities on the two tables.

Figure 1

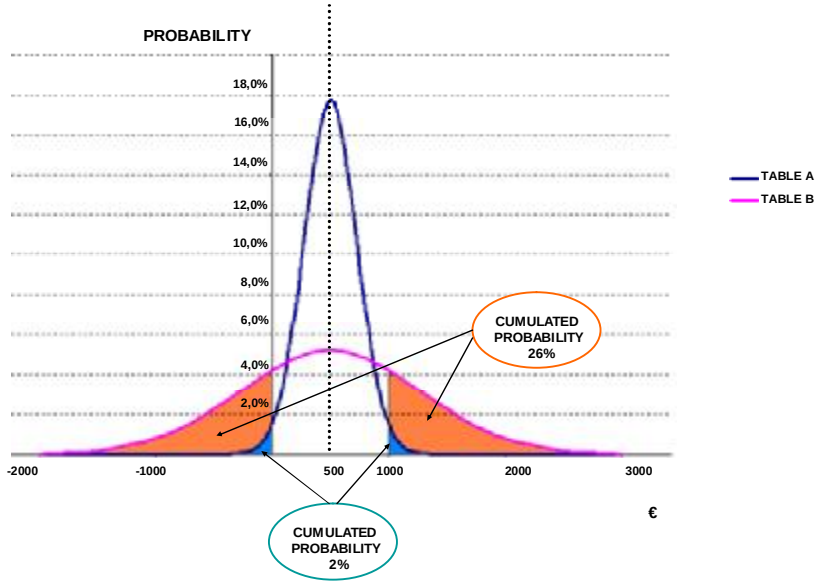


Back to the example of the €5,000 bet on the two tables, based on Figure 1, if you calculate the integral of the two probability curves, you can observe the following, as shown in Figure 2:

On table A, with the 10-number wheel, the probability to close the night with a zero gain or of losing money is about 2%. On table B, with the 100-number wheel, such probability goes up to 26%.

Likewise, on table A the probability to close the night with a gain exceeding €1,000 is 2% while on table B such probability goes up to 26%.

Figure 2



In practice, even though, statistically, each table offers the player a 10% return on the investment (ROI), table B is more risky than table A. If you place 5,000 bets on table B, there is a significant probability not only to gain less than the statistically expected

€500, but, more than that, to lose some of the initial capital. Symmetrically, on the same table there also is a significant probability to gain twice as much or more than the statistically expected €500.

If there existed a Casino with two such tables and rational players, no one would bet on table B.

This is intuitive. In this scenario, a player with €5,000 is about to bet and is faced with two tables, both with an expected gain of 10%; still, since wins on table B are more uncertain and the expected return is the same, players would naturally tend to focus on table A in that it offers more certainty of closing the night with a gain.

In order to attract players to table B, it would be necessary to increase the rate of return so as to compensate the greater uncertainty in the number of wins on a finite series of bets. This could be done, for example, by paying 120 or 130 times the stake rather than 110 as in the example described.

If this were the case, the players would go to one table or the other on the basis of their risk-aversion and of their knowledge of the characteristics and rules of each table. They would choose whether or not to invest, and on which of the two tables to invest, expecting a certain return, being aware that at the close of the night the actual return could be lower or higher.

When it comes to valuating an enterprise or an investment, the very same components are present as those illustrated in the simple example above, i.e.: the initial capital, the expected return, a timeframe over which the investment is analysed and the risk associated to the investment.

In this manual we will see that discounting techniques and the financial theories developed over the last century allow, at least in theory, to take account of all these factors.

With respect to the invested capital, it is useful to further elaborate on the example of the two Casino tables. Assume that some players want to borrow money in order to place their bets.

Obviously, the players are willing to also use their own money but they need to borrow money so as to place more bets and hence gain more.

What happens to the normal distribution of probabilities to gain if we assume that the funds available to players come to the extent of 80% from third-party loans and to the extent of 20% from their own money?

Assume that the lender wants for that night a fixed return equal to 5% of the principal lent. In other words, if a player borrows €4,000, he/she will have to repay €4,200, irrespectively of the outcome of the bets placed. Statistically, every €1 bet financed with a €0.80 loan must return to the lender a €0.04 interest.

Clearly, at the close of the night the gain of the player will be impacted by the obligation to pay the interest on the loan received. However, the player will have committed much less own money to participate in the game and, as a result, will have a higher return.

With respect to the €5,000 initial investment, with an expected return of €500, a player who uses €1,000 of own money to finance him/herself, at the close of the night will have a much higher gain on the investment compared to the previous case:

Without debt:

Opening capital	**5,000 Euro**
expected gain	**500 Euro**
Closing capital	**5,500 Euro**
Return on own capital invested	**10%**

With a €4,000 debt:

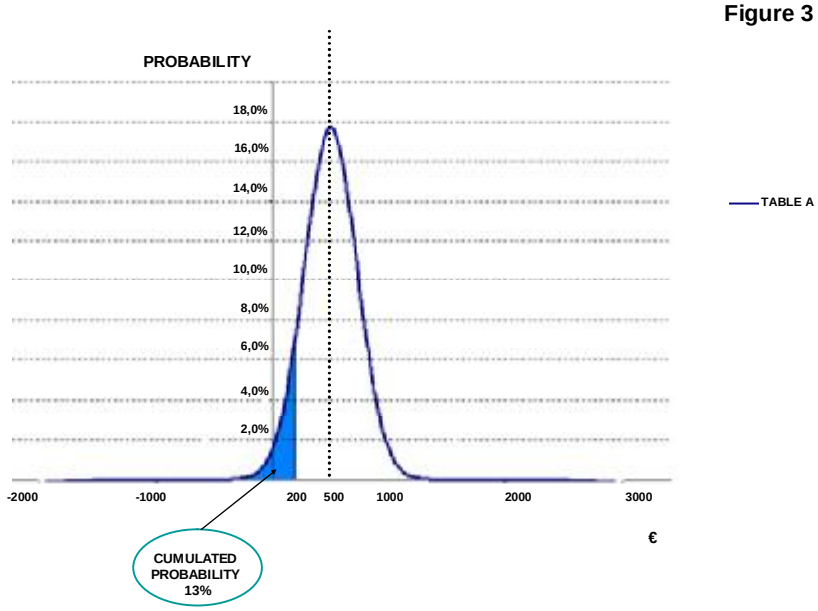
Opening capital	**1,000 Euro**
Loan	**4,000 Euro**
Expected gain	**500 Euro**
Debt and interest repaid	**-4,200 Euro**
Closing capital	**1,300 Euro**
Return on own capital invested	**30%**

Hence, by using debt the player can increase, in % terms, the income-generating capacity of his/her own capital; still, this higher profitability has a side-effect in terms of an inevitably higher investment risk.

Figure 3 shows the normal distribution of the probability to gain on table A as applied to a player who borrows €4,000 to be repaid with €200 of interest whatever the outcome of the bets placed is. As can be seen, on table A the probability to gain less than €200 on the 5,000 bets is 13%.

Opening capital	**1,000 Euro**
Loan	**4,000 Euro**
Actual gain	**< 200 Euro (13% of probability)**
Debt and interest repaid	**- 4,200 Euro**
Closing capital	**< 1,000 Euro (13% of probability)**
Return on investment	**< 0% (13% of probability)**

Figure 3

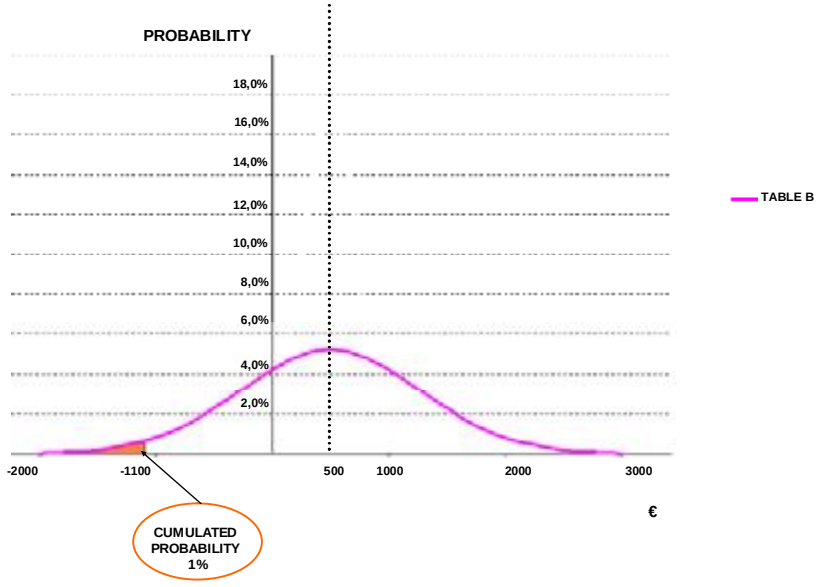


This means that table A, from the viewpoint of a player with borrowed money, is more risky than from the viewpoint of a player sitting at the same table but using solely his/her own money.

The use of a loan to finance bets has added uncertainty, and hence additional risk, compared to the objective risk of table A. In practice, the use of money that must be repaid at a fixed rate of return, shifts risk and concentrates it on the player's own capital.

This does not mean that borrowed money is immune from losses. Figure 4 shows the gain probability of a player on table B financed to the extent of 80% by a loan and to the extent of 20% by own capital. As can be seen, there is a 1% probability that the player incurs losses exceeding €1,100 – or, otherwise said, that he/she eats up the entire capital.

Figure 4



In this unfortunate case, the player will not be capable of repaying the entire loan plus interest thereon.

Opening capital	**1,000 Euro**
Loan	**4,000 Euro**
Actual gain	**< -1,100 Euro (1% probability)**
Debt and interest repaid	**-4,200 Euro**
Closing capital	**< -100 Euro (1% probability)**

In other words, also the fixed-rate-of-return player suffers, though to a lesser extent, from the player's risk. This happens because it might turn out to be impossible to repay the entire loan plus interest thereon, unless, of course, the player contributes with more own capital to top up the capital shortfall at the close of the night. This risk can lead the lender to increase the interest rate or reduce the amount lent to the player. The behaviour of financial institutions is perfectly in line with this principle.

In a world of rational individuals and where each player is aware of the probabilities of tables A and B, the behaviour of money lenders would follow in the track of that of players. The money lent to a player who bets on the 100-number wheel is more prone to risk: as a result, the lender will apply a higher rate of interest. This is because the normal distribution of probabilities of table B is broader compared to that of table A.

Ultimately, the risk aspects of an investment impact the returns for those that provided the finance to such an investment - be they shareholders or banks.

An investor who is about to purchase an enterprise or implement a project, is entirely exposed to the risk characteristics of the industry in which the investment is made. Furthermore, in turn, the investor can take the additional risk arising from the bank loan. The total level of risk to which the capital invested by the shareholder is exposed is the result of the industry risk amplified by the financial leverage.

3. Representative Methodology

Prior to explain the details of the various calculation systems, it is useful to briefly summarise the representative methodology as well as the terms traditionally used in the business and financial world.

When valuating an enterprise or a project, the analyst builds a business model that simulates the P&L, the balance sheet and the cash flow statement.

When building a business model, bear in mind that you are trying to simulate the behaviour of an investment over a given period of time. Such behaviour is ultimately ascribable to the accounting movements and to the daily cash flows in and out of the corporate current account.

For the sake of convenience, a business model cannot reflect the daily life of the enterprise: this would be incredibly complicated and unnecessary. A model simulates the behaviour of the enterprise by breaking it down into finite periods (e.g., years or months).

Most important, bear in mind the basic assumption of all business models: all cash movements and P&L movements occur at the very moment each finite period ends, while all balance sheet items remain unaltered over one specific period.

In modelling, a high-level recap needs be at hand at all times: indeed, this gives an overview of the overall situation of the investment and spots the elements that will be subsequently used for discounting calculations. Also, always remember to keep things simple and avoid overcomplicating the picture.

Below is an illustration of the basic layout of a valuation model. It is understood that behind each line of the P&L and of the balance sheet there can be calculations, sometime even very complex ones, relating to the factors that impact the trend of the phenomenon. For example, sales are connected to product quantities, prices, market shares, etc. However, such calculations can be done on another spreadsheet, of which only the line showing the sales is incorporated in the valuation model. This applies to any element of the P&L, of the balance sheet or of the cash flow statement.

Below is a short description of the 3 sets of accounts required to work out a valuation model:

3.1 Profit & Loss Account (P&L)

The short form is comprised of 9 elements.

a) Revenues, i.e. all of the enterprise's proceeds from the sale to third parties of goods and services.
b) Costs relating to the generation of revenues.
c) The EBITDA. Difference between revenues and costs.
d) Amortisation and depreciation that can be deducted for tax purposes.
e) The operating profit or EBIT. Difference between the EBITDA and amortisation & depreciation.
f) Bank interests, either positive of negative.
g) Earnings before tax. Difference between the operating profit and net interests.

h) Taxes in respect of which the interests are deductible for tax purposes. All the other taxes must be treated as pure costs.
i) Earnings after tax or EAT.

3.2 Balance Sheet

In short, the balance sheet is comprised of 4 elements.

a) Tangible and intangible assets, subject to depreciation and amortisation respectively, also referred to as Fixed Assets.

b)Working Capital, i.e. all of the items of the balance sheet that ensure that the enterprise can operate. The working capital does not comprise any financial element, but only inventories, payables, receivables and deferrals and accruals vis-à-vis third parties excluding banks and shareholders. Therefore, the working capital also includes the severance indemnity, V.A.T. and corporate tax payables and/or receivables.

d) Banks debts net of any cash shown in the balance sheet.

e) Shareholders' equity including reserves and any other item referred to as equity.

3.3 Cash Flow Statement

The cash flow statement is the third of the three accounts needed to represent an enterprise or a project and is of fundamental importance in that it reconciles the P&L and the balance sheet.

The short form of the cash flow statement is comprised of 7 elements:

a) EAT derived from the P&L for the period.
b) Amortisation & depreciation derived from the P&L for the period.
c) Investments made by the enterprise in the period.
d) Changes between opening and closing working capital.
e) Contributions of capital by shareholders in the period.
f) Dividends or distributions of capital paid to shareholders in the period.
g) Change in the financial position arising from the previous items and shown in the balance sheet as net debt to banks.

The representation of any project or enterprise can be referred to the short list of this chapter. The analysis and examples in this manual refer to this layout.

The modelling of an enterprise or a project consists in the series of P&Ls, balance sheets and cash flow statements developed over a given period of time. The business model of a project could simulate the entire business life of the project. The business model of an enterprise could simulate the mere time frame during which the shareholders intend to maintain the ownership until the decision is made to transfer their shareholdings. Normally, a business model covers a time span of 5 or 10 years and establishes a terminal value downstream of the last year.

The terminal value could be that of re-selling the enterprise or putting it in liquidation or its calculated assuming it continues operation for an undefined period of time. These are scenarios that the analyst must consider on a case-by-case basis.

The business model is not only influenced by the characteristic decisions of the project or of the enterprise to be analysed but also by the macro-economic environment in which it develops. The most important variable for these purposes is inflation.

In practice, zero-inflation business models (i.e., expressed in real money at the time the valuation is made by the analyst) are quite common to be found. This practice, though broadly used, is technically incorrect.

Indeed, in a real-money model, one tends to overestimate the value of amortisation & depreciation compared to other items. Amortisation & depreciation are constant and depend upon the previous investments; therefore, over time they tend to have more weight than the other items of the P&L. In point of fact, amortization & depreciation would have less weight in a P&L subject to inflation. Over a ten-year time frame, such discrepancy can be significant.

Therefore, it is suggested to build business models in nominal money, i.e. factoring in inflation. However, in Western economies, where the inflation rate attains 1-2%, a real-money business model is acceptable. It all depends upon the analyst's sensitivity.

Every chapter of this manual refers to an archive that can be downloaded from the website under the name *spreadsheet.xls* which, in turn, contains multiple sheets.

The name of the reference spreadsheet used to explain the methodologies is indicated in the heading of each chapter.

4. Terminology

This manual uses English terms that are commonly used worldwide. These terms are the same to be found in US or UK academic literature and are the reference jargon of this work.

For the sake of convenience, they are briefly summarised below.

βa, βassets: index of volatility of the cash generated by an investment in a given industry.

βe, βequity: index of volatility of the cash generated for shareholders by an investment in a given industry using a given debt component.

EAT (Earning after tax).

EBIT (Earning before interests and tax).

EBITDA (Earnings before interests, tax, depreciation and amortization).

EBT (Earnings before tax).

Equity: the net worth of the enterprise, being the value that the enterprise represents for its shareholders. It can be at book value, in which case it is represented by the total shareholders' equity shown in the balance sheet, or at market value, in which case it is represented by the market value of shares. In the absence of market values, the market value of the equity is defined as the value obtained by discounting the future cash flows that the shareholders will receive from the enterprise.

EVA (Economic value added): it is the value rendered to shareholders over a given period of time in addition to what the shareholders expected in the same period as a function of the investment risk.

Free Cash Flow: in the classical terminology it is the free cash flow for shareholders as determined using the WACC methodology, i.e., assuming that the EBIT is fully taxed. In point of fact, the real cash flows to shareholders are very different.

GIC (Gross invested capital). It is equal to the sum of investments, gross of amortisation & depreciation, plus working capital.

Kd: cost of the money provided by banks or by financial institutions to finance the investment.

Ke: cost of equity requested by the shareholders who finance an investment.

NIC (Net invested capital). It is equal to the sum of investments, net of amortization & depreciation, plus the working capital.

NOPAT (Net operating profit after tax). In practice it is the EBIT fully taxed.

NPV (Net present value) or also PV (present value): traditionally, this expresses the discounted value of any one series of cash flows. It commonly refers to the cash flows received by shareholders. It is net of cash outflows, if any. It refers to the moment

when a project is created, or when the decision is made to implement it. In the valuation of an enterprise, the PV refers to the cash flows that the enterprise will generate in the future while the NPV refers to the difference between the PV of future cash flows and the investment that the shareholders make to purchase it.

P&L (profit and loss account).

T: taxes on corporate income. These are the taxes paid by the enterprise, in respect of which the interest is deductible for tax purposes. In Italy, this tax is referred to as IRES. These taxes are not to be confused with the total taxes paid by the enterprise. The taxes that are not impacted by the cost of money are to be considered as pure costs and must be treated as such in the business model. In Italy, the taxes to be considered as costs are ICI or IRAP, in respect of which the cost of money is not fiscally deductible.

TEP (Total enterprise value). At any point in time, the total enterprise value or the total value of a project can be expressed as the sum of equity value and debt value.

Working Capital. This is the sum of all those balance sheet items - be they assets or liabilities - that have these characteristics: items other than depreciable/amortisable tangibles or intangibles assets, items other than bank debts, items that do not fall under the definition of equity and items other than financial investments made by the enterprise, which allow the enterprise to operate. Typically, the working capital is comprised of the following items, most of them are short-term but some are long-term: inventories, trade receivables, trade payables, accruals & deferrals, severance pay, V.A.T. payable and receivable, tax payables and receivables generated by the operations of the enterprise.

5. Time Value of Money

As mentioned in the Introduction, this manual addresses those who wish to delve into the methods for valuating an enterprise or a project with the discounted cash flow (DCF) technique.

Any economic system intended to create added value and profits rests on projects. Projects are entrepreneurial activities aimed to generate cash flows exceeding the investments made.

The stereotype of a project is composed of a number of activities that absorb cash upfront and generate cash during its useful life. The same definition, in a broader sense, applies to an enterprise, which is but a set of projects the life spans of which overlap, thus generating a continuum of cash flows.

In practice, if you wish to determine the value "as of today" of a series of future cash flows rendered to shareholders, you need to discount them at a given discount rate that reflects both the risk of the project and the allocation of such risk amongst shareholders and banks.

The discount rate depends upon the risk of cash flows: the higher the risk, the higher the discount rate. A given sum of money, to be received in the future, has a present value "today": the higher the risk or uncertainty of the activity that will generate it, the lower this value.

Likewise, the higher the risk of an investment, the higher the rate of return expected by those who will finance the initiative. As a result, if you invest today a given sum of money in a low-risk activity, you expect a fairly low rate of return, whereas, if the activity were more risky and uncertain, you would expect a proportionally higher return.

Please note that by risk it is meant the objective risk of a given activity: i.e. the risk that cannot be eliminated or reduced because it is typical of that activity. In point of fact, in daily life, it is human beings who make the decisions: for this reason, the risk perceived by the individuals who are about to valuate an initiative does not necessarily reflect the objective risk of that activity, and this can lead to wrong valuations and decisions.

For example, an entrepreneur focusing on the construction industry could perceive as being very risky an investment in an industry he/she is not acquainted with, like, for example, the transportation industry, for the simple fact that he/she does not know much about that industry. The financial market does not recognize and does not valuate this type of risk because the entrepreneur can hire specialised consultants and managers who, by contributing their knowledge, can cross out subjective bias.

Likewise, the same entrepreneur, possibly driven by his/her own knowledge of the construction industry, might underestimate the objective risks of the industry and invest in the wrong projects, thus exposing him/herself to losses or returns lower than those normally associated to the level of risk of the industry in which he/she operates.

Here again, the financial market does not allow for any risk mitigation arising from a skilful entrepreneur or management. The management skills impact directly on the higher cash flows generated by the enterprise. However, the cash so generated will in any case have to be discounted taking account of the proper level of risk of the industry, which does not depend upon the skills of those who manage the enterprise.

No doubt the enterprise value will benefit from a skilful management. Still, the industry risk is the same and typical for all the enterprises operating in the same field.

Industry risk is evaluated and interpreted assuming that all can do their job and that all have the same information. In real life, this is not so; however, it is up to the expert financial analyst to first apply theoretical notions and then mediate and correct them in light of any possible peculiarity as may arise from the case he/she is analysing.

6. Quantification of Risk

The purpose of this chapter is to summarise the various notions developed in Finance for quantifying the risk associated to an expected cash flow. The reference theory is the Capital Assets Pricing Model (CAPM).

The principles set out here apply to projects and enterprises alike, as will be shown later.

The purpose of a project is to generate cash flows from the investments made in entrepreneurial activities. In practice, a project is a set of activities that absorb cash upfront and generate cash during its useful life.

The cash flows generated by the project fall into three categories:

- **cash flows to banks, to serve and honour the debt used to finance the project.**
- **cash flows to shareholders for the capital contributed by shareholders to finance the project.**
- **cash flows to tax authorities, in that an activity aimed to generate profit cannot, of course, avoid paying taxes.**

The risk associated to the cash flows generated by the project is a risk that, as said earlier, depends upon the industry and the country in which the project is implemented. Such risk is quantified with the so-called β assets (βa).

βa is a number, higher or lower than 1, that is measured by various financial institutions and is expressed as the relative volatility of the return of a given investment compared to the market. βa can be purchased in the form of reports and refers to entire industries (tlc, chemistry, construction etc.).

βa is a number that incorporates the distribution of an industry's income-generation probabilities.

Since the cash generated by the project is to be used to repay banks, shareholders and tax authorities, the risk associated to the cash flows of the project must be reflected in part in the risk of the cash flows to be used to serve bank debt and in part in the risk of the cash flows to be used to repay shareholders and tax authorities.

We know that in a project the service of bank debt takes priority over shareholders; also, remuneration conditions are independent from the industry performance. Therefore, generally and put simply, it can be stated that the cash flows to be used to repay debt are significantly less risky than the share capital, i.e., they are not volatile. They are not subject to a bell shaped distribution of probabilities, i.e., the interest payable is pre-determined and unvaried. The cost of money has a β of zero.

As a matter of fact, this the reality. Once a long-term loan has been negotiated to finance an enterprise or a project, the interest rate only changes depending upon the macro-economic scenario or upon the interest rate established by the central bank, in that all of the other elements are established in the contract entered into by and between the enterprise and the bank.

Kd is normally anchored to the reference bank rates, such as Euribor, plus a spread that is negotiated with the bank that acts as lender. Kd is in all cases in nominal terms, i.e. gross of the inflation.

If the debt's β is zero, it follows that the entire βa must be reflected onto βequity (βe).

βe represents the risk of the cash flows received by shareholders and tax authorities. The latter are treated like shareholders as they only receive money after bank debt has been repaid.

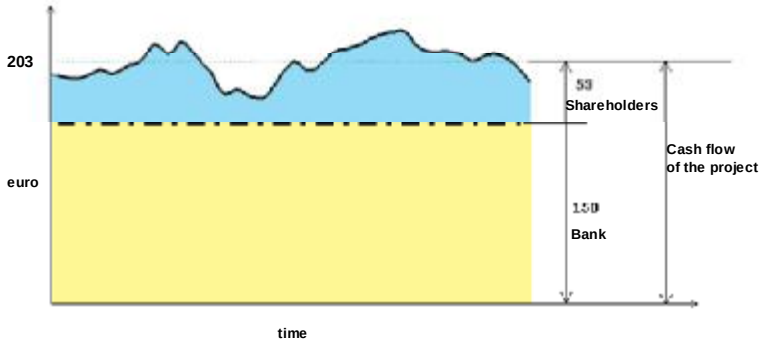
Obviously, where debt is zero, βa equals βe, i.e., the industry risk is mirrored in Equity risk. Instead, where debt exceeds zero, the shareholders' risk inevitably increases.

Figure 5 shows the chart relating to the cash generation of a project over a given period of time (bold dotted line).

In this example, the project generates an average cash flow of €203 per reference period (usually one year). Out of these, €150 per year are used to repay the debt (yellow area within the bold dotted line) and an average €53 per year are repaid to shareholders and tax authorities (light dotted blue line).

Clearly, the risk of cash flows to shareholders increases as debt increases.

Figure 5



The standard deviation of the cash flows generated by the project is €14.50, i.e., there is a 66% probability that in a given year the cash generated by the project ranges between 203 -14.50 = 188.50 Euros (floor) and 203 + 14.50 = 217.50 Euros (ceiling).

Standard deviation measures the width of the normal distribution of events and can be expressed as a % of the mean of results.

In our case, the project shows a standard deviation of 7% of the mean of cash flows generated by the project to serve banks and shareholders:

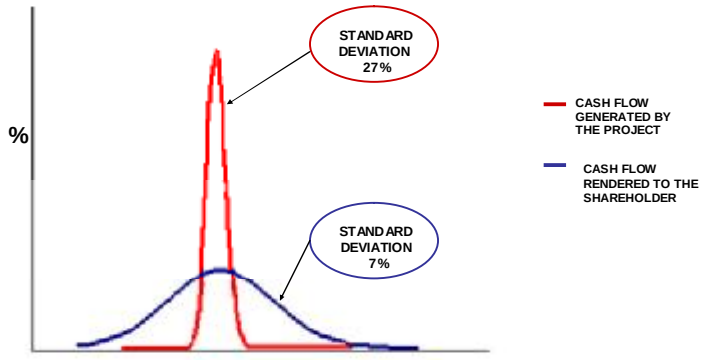
14.50/203 = 0.07 = 7%

The very same standard deviation, though, attains 27% if expressed in relation to the mean of cash flows to shareholders:

14.50/53 = 0.27 = 27%

The chart under Figure 6 shows the distribution of probabilities of the project cash flows (7% of the mean of results) and of the cash flows to shareholders (27% of the mean of results)

Figure 6



Clearly, in a given project with a level of risk that is typical of the relevant industry, as indebtedness increases, the bell curve of probabilities of cash flows to shareholders is broader compared to the cash flows generated by the project.

This means that, as indebtedness increases, the return to shareholders can also be much higher or much lower than the same unlevered project. There is wide historical evidence showing that big gains or big failures for enterprises and their shareholders are also linked to their level of indebtedness.

The mathematical formula that links equity risk to the size of debt is as follows:

$\beta e = \beta a \cdot F$ **where F = financial leverage factor**

$F = 1 + (1-T) \cdot D/E$ **where T = corporate income tax**
D/E = project Debt/Equity ratio

The corporate income tax impacts the risk of cash flows to shareholders in that debt generates interest, which, in turn, drives down taxable income. In other words, the tax shield generated by the cost of debt mitigates the increase in the risk for shareholders that arises from the debt undertaken.

In the literature and in financial reports βa is at times referred to as βeU (Equity unlevered, i.e. without debt). Instead, the βe corresponding to a level of indebtedness higher than zero is referred to as βeL (Equity levered, i.e. with debt) and it is typical of a given enterprise with a given level of indebtedness.

The meaning is the same and stems from the way in which β is captured and calculated. In practice, when a financial institution calculates β, it uses as primary source the values of the stocks of enterprises listed on the Stock Exchange and operating in a given industry of a given market, e.g. the chemical industry in the U.S.A., and calculates over a given period the βeL of each enterprise operating in such industry.

Obviously, the βeL of a given enterprise is impacted by its level of indebtedness; hence it needs be "corrected" so as to cross out the effect of debt on the stock risk:

βeU = βeL/(1+(1-T)·D/E)

This calculation is done directly by the issuer that publishes for each enterprise the βeU obtained by calculating the βeL measured on the market and then corrected by T (corporate income tax) and D/E (the ratio between financial debt and equity as measured on the stock market).

The mean of the βeUs of the enterprises of a given industry is referred to as the industry βa.

On the basis of D/E, βa and taxation you can determine the βeL of an investment, which, in turn, is used to determine the cost of equity, i.e. the expected return for shareholders.

From now on, when mentioned, βe refers to βeL and it is used to determine the cost of Equity:

Ke = Rf + Rp·βe

Ke is the cost of share capital, i.e. the interest rate expected by the shareholders who invest in the enterprise under those given conditions.

Rf is the risk-free market rate, i.e. the rate of return of government bonds. Rf is derived directly from public sources of financial information and normally refers to long-term rates. Rf can be expressed in nominal or real terms, depending on whether inflation is factored in; published data always refers to nominal Rf.

Rp is the stock market risk premium, i.e. the premium that investors expect from a risk investment as is the case with investments in stocks. Rp is periodically measured by financial institutions or universities as part of academic works. The value of Rp can be expressed in real or nominal terms, as specified by the issuer. Currently, 6% is used for investments in the European and American markets alike. In general, however, the Rp values mentioned in the literature range between 5 and 8% in nominal terms.

The Ke derived from the calculation using nominal Rf and nominal Rp is referred to as nominal Ke and represents the expected return for shareholders, including the inflation, on an stock investment with a given risk profile that depends upon the industry and the level of indebtedness of the project.

Ke captures all of the elements of risk from the point of view of the shareholders who participate in a given investment.

Without discounting the cash flows to shareholders with the Ke determined as above, it would be very hard to valuate a project. In fact, you would need to work out a curve of the distribution of probabilities of gain, year by year, and then cumulate it. Ultimately, you would have a very complex amount of data to be both interpreted and communicated.

Instead, by discounting the cash flows for shareholders using Ke, you already factor in all of the necessary elements. As consequence the future cash flows discounted as of today take into account the probability of their materialising.

The formulas shown in this chapter can be expressed in nominal or real terms, depending on whether you include inflation. Obviously, you can at any time shift from a real to a nominal rate, as shown in the examples below:

nominal Ke = (1 + real Ke) · (1 + i) – 1

real Kd = (1 + nominal Kd) / (1 + i) -1

real Rf = (1 + nominal Rf)/(1+ i) -1

where i = inflation expected in the very period to which the valuation business model refers.

7. Definition of Debt and Equity at Market Values

When dealing with β mention was made of the D/E ratio, where D is the enterprise debt and E is its equity.

A correct interpretation of the definition of D and E is fundamental to understand this manual and for a correct application of the principles set forth therein.

By debt it is meant the money provided by banks to the project or to the enterprise. Debt is repaid before repaying shareholders. Debt is shown in the balance sheet and is a specific value referring to a specific point in time, e.g. debt as at 31 December.

Besides being derived from the balance sheet, debt can also be determined by discounting the cash flows to banks at a discount rate equal to the market rate.

Example:

Let us verify that a €1mn debt as at 31.12.2000 at an interest rate of 4% p.a. for 5 years, with €1mn repaid in year 5, leads to a discounted value of €1mn:

		31/12/2000	31/12/2001	31/12/2002	31/12/2003	31/12/2004	31/12/2005
Interests paid to the bank	k€		40,0	40,0	40,0	40,0	40,0
Principal pay back to the bank	k€						1.000,0
Total cash flows to the bank	k€		40,0	40,0	40,0	40,0	1.040,0
Discount factor at interest rate	4%	1,00	0,96	0,92	0,89	0,85	0,82
Discounted cash flows to the bank	k€		38,5	37,0	35,6	34,2	854,8
Present value of cash flows to bank	k€	1.000,0					

So long as the cost of money applied by the bank to the enterprise for calculating interest is equal to the market rate, the value of debt shown in the balance sheet equals that calculated with the discounting method. In the most commonly used valuation models, this is the principle normally applied and hence D is the book value of debt at a given point in time.

There can be special cases, like for example loans granted at rates lower than market rates: in this case, the value of debt shown in the balance sheet does not correspond to the real financial value and this needs be considered in discounting calculations.

Example:

A €1mn debt granted on 31.12.2000 at an interest rate of 2% p.a., compared to a market rate of 4% p.a., for 5 years, with €1mn repaid in year 5, leads to a discounted value of €911k.

		31/12/2000	31/12/2001	31/12/2002	31/12/2003	31/12/2004	31/12/2005
Interests paid to the bank	k€		20,0	20,0	20,0	20,0	20,0
Principal pay back to the bank	k€						1.000,0
Total cash flows to the bank	k€		20,0	20,0	20,0	20,0	1.020,0
Discount factor at interest rate	4%	1,00	0,96	0,92	0,89	0,85	0,82
Discounted cash flows to the bank	k€		19,2	18,5	17,8	17,1	838,4
Present value of cash flows to bank	k€	911,0					

This can be intuitively understood. Imagine that the €1mn granted at a facilitated rate of 2% by the bank is, in turn, lent by the enterprise to a third party at the 4% market rate. This entails, for the enterprise and for its shareholders, an interest income of €20k p.a., equal to the difference between the rate paid by the third party and the facilitated rate applied by the bank. At the end of year 5, the third party will give the money back to the enterprise, which, in turn, will give it back to the bank.

Below is the table of cash flows, which shows that the enterprise would receive €89k of net discounted interest income, which would drive debt down from €1mn to €911k, as said earlier.

		31/12/2000	31/12/2001	31/12/2002	31/12/2003	31/12/2004	31/12/2005
Interests from third parties	k€		-40	-40	-40	-40	-40
Interests paid to the bank	k€		20	20	20	20	20
Principal pay back to the bank	k€						0
Total cash flows to the bank	k€		-20	-20	-20	-20	-20
Discount factor at interest rate	4%	1,00	0,96	0,92	0,89	0,85	0,82
Discounted cash flows to the bank	k€		-19,2	-18,5	-17,8	-17,1	-16,4
Present value of cash flows to bank	k€	-89,0					

These are, however, a fairly rare occurrence. Indeed, the assumption here is that the bank can recover from other sources the difference between the market rate and the facilitated rate. This occurs with subsidized loans provided for by government policies.

These principles applies to the Equity as well. Traditionally, the equity value is the market value. It should, however, be borne in mind that, in contrast with debt, the difference between the book value of equity and its market value can, at times, be very significant. For sure, it is fairly unusual for book value and market value to be the same for long periods of time.

Unfortunately, for the vast majority of enterprises, there is no immediate way to determine their equity market value in that only a small percentage of them are listed on the Stock Exchange. It is therefore necessary to identify an equity valuation method that compares to market value.

It is widely known that the most rational method to determine the equity value is discounting cash flows to shareholders. This method applies to listed and unlisted enterprises alike as well as to investment projects.

It may well be that the Stock Exchange value of shares does not correspond to the value calculated by discounting the cash flows to shareholders; you should not, however, be mislead by this. The share price is subject to many short-term phenomena (related to either speculation or psychology) which may cause temporary differences, including significant ones, compared to the value calculated by discounting cash flows.

Despite market distortions, the leading financial institutions and the most reliable analysts broadly use the discounted cash flow method to determine the share value. For this reason, this method turns out to be the most well-grounded one in business and financial terms as well as the one most commonly used.

The correct determination of the equity value of a project or of an enterprise is one of the main technical topics of this manual.

8. Cash to Equity Methodology with Variable D/E

Refer to "Project ALFA" in the Spreadsheet.XLS file

Assume that you need to valuate project Alfa, which has the following features:

- **year 1 investment of 350;**
- **3 years of cash flows generated by the project from year 2 to year 4;**
- **project to be liquidated at the end of year 4 with the collection of the book value of the working capital assuming that the market value of amortised and depreciated assets is zero;**
- **the share capital initially provided by shareholders is 150, while the remaining part of the investment is financed by banks;**
- **the initial investment is followed by other investments in assets in subsequent years so as to maintain income generation;**
- **the working capital has the year-by-year profile described in the balance sheet.**
- **the EAT is always entirely distributed.**

The fundamental valuation parameters are shown in the project Alfa spreadsheet in the yellow Input Zone of the business model:

1	**INPUT ZONE**						
2							
3	Kd	Input	8,0%				
4	Rp	Input	6,0%				
5	Rf	Input	5,0%				
6	Taxes	Input	50,0%				
7	Beta assets	Input	1,000				
8	Terminal value	Input	Liquidation of working capital				
9							
10	**P&L**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
11							
12	Revenues	Input		0	600	1.000	1.000
13	Costs	Input		0	-250	-750	-750
14	Depreciation	Input		0	-150	-150	-150
15							
16	**CASH**						
17							
18	Equity contribution	Input		150	0	0	0
19	Investments	Input		350	50	50	0
20							
21	**BALANCE SHEET**						
22							
23	Gross fixed assets	Input	0				
24	Cumulated depreciation	Input	0				
25	Working capital	Input	0	40	100	150	150
26	Debt	Input	0				
27	Share capital	Input	0				

Below is the business model of the project. All values are in nominal currency year by year, i.e. they factor in inflation:

41	**P&L**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	
42							
43	Revenues		0,0	600,0	1.000,0	1.000,0	
44	Costs		0,0	-250,0	-750,0	-750,0	
45	EBITDA		0,0	350,0	250,0	250,0	
46	Depreciation		0,0	-150,0	-150,0	-150,0	
47	EBIT		0,0	200,0	100,0	100,0	
48	Interests		0,0	-19,2	-16,0	-12,0	
49	EBT		0,0	180,8	84,0	88,0	
50	Taxes		0,0	-90,4	-42,0	-44,0	
51	EAT		0,0	90,4	42,0	44,0	
52							
53							
54	**BALANCE SHEET**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
55							
56	Gross fixed assets	0,0	350,0	400,0	450,0	450,0	450,0
57	Cumulated depreciation	0,0	0,0	-150,0	-300,0	-450,0	-450,0
58	Net Fixed Assets	0,0	350,0	250,0	150,0	0,0	0,0
59							
60	Working capital	0,0	40,0	100,0	150,0	150,0	0,0
61							
62	TOTAL ASSETS	0,0	390,0	350,0	300,0	150,0	0,0
63							
64	Debt	0,0	240,0	200,0	150,0	0,0	0,0
65	Share capital	0,0	150,0	150,0	150,0	150,0	0,0
66	TOTAL LIABILITIES	0,0	390,0	350,0	300,0	150,0	0,0
67							
68	*check*	0,000	0,000	0,000	0,000	0,000	0,000
69							
70	**CASH FLOW STATEMENT**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
71							
72	Sources of funds						
73	EAT		0,0	90,4	42,0	44,0	0,0
74	Depreciation		0,0	150,0	150,0	150,0	0,0
75	Equity contribution		150,0	0,0	0,0	0,0	0,0
76	**Total sources**		150,0	240,4	192,0	194,0	0,0
77							
78	Uses of funds						
79	Investments		350,0	50,0	50,0	0,0	0,0
80	Increase in working capital		40,0	60,0	50,0	0,0	-150,0
81	Dividends and capital distribution		0,0	90,4	42,0	44,0	150,0
82	**Total uses**		390,0	200,4	142,0	44,0	0,0
83							
84	**Sources minus uses of funds**		-240,0	40,0	50,0	150,0	0,0
85							
86	Net cash position beginning of period		0,0	-240,0	-200,0	-150,0	0,0
87	Sources minus uses of funds		-240,0	40,0	50,0	150,0	0,0
88	Net cash ending of period	0,0	-240,0	-200,0	-150,0	0,0	0,0

The cash flow statement shows the cash flows generated by the project and those destined to - or requested from - shareholders. Year 0, i.e. when the decision is made as to whether or not to make the investment, is 31.12.2000: all future cash flows are discounted to this date.

2001 is year one, when, on 31 December, the share capital is provided, the investment is made and part of the working capital is established. The total requirement is 390, of which 150 funded by shareholders and 240 with bank debt, which represents the total net amounts owed to banks at 31.12.2001.

The financial position at the end of 2001 is assumed to be constant throughout 2002 and generates interest expense of 19 in that the cost of money is 8% p.a. The financial position only changes at the end of each year on the basis of the cash flows indicated by the cash flow statement and remains constant for the next following year and so on for 2003 and 2004.

The project stops generating cash at the end of 2004 and is liquidated immediately thereafter. For the sake of simplicity and clarity, liquidation is shown on 1.1.2005 even though it takes place at the end of year 4.

Let us now move on to the calculation of the equity value of the project on the basis of the cash flows to shareholders and on the basis of the Ke year by year to be used to discount such cash flows.

This method, as summarized in the table below, is referred to as Cash to Equity Methodology with Variable D/E year by year:

90	**Cash to Equity methodology with variable D/E year by year**						
91							**Liquidation**
92	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
93							
94	Debt at liquidation						0,0
95	Equity at liquidation						150,0
96	Total enterprise value at liquidation (D+E)						150,0
97	Debt at beginning of period		0,0	240,0	200,0	150,0	0,0
98	Cash to shareholders:						
99	(dividends + capital distributions - equity contribution)		-150,0	90,4	42,0	44,0	150,0
100							
101							**Liquidation**
102	**Determination of D/E, F and Ke year by year**		**2001**	**2002**	**2003**	**2004**	**2005**
103							
104	Kd		8%	8%	8%	8%	
105	Rp		6%	6%	6%	6%	
106	Rf		5%	5%	5%	5%	
107	Taxes		50%	50%	50%	50%	
108	F		1,00	1,49	1,54	1,44	
109	Beta assets		1,00	1,00	1,00	1,00	
110	Beta equity		1,00	1,49	1,54	1,44	
111	Ke		11,0%	14,0%	14,2%	13,6%	
112							
113	PV of Equity at the beginning of each period		**83,5**	242,7	186,2	170,7	
114							
115	D/E of each period		0,00	0,99	1,07	0,88	0,00
116							
117	Note: the value of the equity at the beginning of one period is calculated by discounting one						
118	year the cash flow to shareholders at the end of the period together with the value of the						
119	equity calculated for the following period with the same methodology						
120							
121	Total enterprise value (D+E) at beginning of the period		83,5	482,7	386,2	320,7	

The calculation starts from the last year of the project and goes back, year by year, to the beginning, i.e. to year 0, which typically is the time the decision is made and when shareholders want to know the value of their equity.

If you trace back the calculation starting from the last year, you can ensure that the Ke of a given year is consistent with that of the years following thereafter in that it expressly incorporates them.

Start from the last point in time of the project, i.e. liquidation. On that date, the end of year 4, the residual debt is zero and cash receipts from the liquidation of the working capital stand at 150.

The shareholders will receive all of the proceeds from liquidation and, at the close of year 4, they will also receive dividends of 44, totalling 194.0 as at 31.12.2004.

The debt valid for 2004 is shown in the balance sheet. It equals debt as at 31.12.2003, maintained constant throughout 2004 and then cancelled on the basis of the cash generated in the year, as verifiable in the cash flow statement.

Obviously, the value of debt at the beginning of 2004 can also be verified with the discounting methodology:

$(D)_{2004}$ = (150.0 + interest paid in 2004)/(1+Kd) = (150.0+12.0)/(1+0.08) = 150.0

The equity value at the beginning of 2004 is the result of discounting 194.0, received by shareholders at the end of 2004, using a discount factor of 1/(1+Ke), where Ke is that of 2004.

Since Ke depends upon D/E, and since we know D, the following equations need be solved:

E = (150.0 + 44.0) / (1+Ke)

D/E = 150/E = 150· (1+Ke)/(150+44) = 0.773· (1+Ke)

Then F is solved as a function of Ke:

F = 1 + (1-T) ·D/E = 1 + (1-0.5) ·0.773· (1+Ke) = 1 + 0.387· (1+Ke)

then, we make βe explicit as a function of Ke:

βe = βa·F = βa· (1 + 0.387· (1+Ke)) = 1 + 0.387· (1+Ke)

finally, we solve Ke:

Ke = Rf + Rp ·βe = 0.05 + 0.06· (1 + 0.387· (1+Ke)) = 0.11 + 0.023· (1+Ke)

Ke – 0.023·Ke = 0.133

Ke = 0.133/(1-.023) = 0.136 i.e., 13.6%

hence:

E = 170.7
D/E = 0.88

The complete formula to solve Ke, to be included in the cells of line 111 of the model is as follows:

$$(Ke)_n = (Rf+Rp\cdot\beta a+Rp\cdot\beta a\cdot (D)_n\cdot(1-T)/((Div)_n+(E)_{n+1})/(1-Rp\cdot\beta a\cdot(1-T)\cdot (D)_n/((Div)_n+(E)_{n+1})$$

where:

$(Ke)_n$	**Cost of equity applicable to year *n***
$(D)_n$	**Debt at the beginning of year *n* and constant throughout the period**
$(Div)_n$	**Dividends or capital payouts received by shareholders at the end of year *n***
$(E)_{n+1}$	**Value of equity at the beginning of year *n+1*, same as at the end of year *n***

Then, move on to apply the same method to year 3. In this case we no longer have the liquidation value but we have the equity value at the beginning of year 4 (as calculated shortly ago).

At the end of year 3 the shareholders receive dividends of 42.0, in addition they also receive 170.7, i.e. the discounted cash flows paid at the end of year 4. The debt at the beginning of year 3 is 200.0 and is constant throughout the year.

Hence:
Ke = 14.2%
E = 186.2
D/E = 1.07

The equity value at the beginning of year 3, of 186.2, factors in the discounted cash flows from liquidation at the end of year 4 and the cash flows from dividends at the end of year 3 and 4. Ke at the beginning of year 3 is therefore consistent with years 3 and 4. The same holds true for D/E.

This procedure is repeated for year 2, at the end of which the shareholders receive dividends of 90 coupled with 186.2 of discounted cash flows received starting from year 3. Throughout year 2 Debt stands at 240.

Hence:
Ke = 14.0%
E = 242.7
D/E = 0.99

Finally, move to year 1 of the project. At the end of year 1 the shareholders do not receive any dividends and provide 150 of capital, to be subtracted from the equity value of 242.7 as calculated at the beginning of year 2. The capital contributions paid in are negative compared to dividends in that these are flows from shareholders to the enterprise and not the other way round.

The cash flow statement shows that debt at the beginning of year 1 is zero in that it is entirely paid at the end of the period. Therefore, D/E is zero and Ke is 11%. It follows that equity at the beginning of year 1 is 83.5.

83.5 is the net discounted value of cash flows to shareholders (net present value, NPV). The value has been discounted on the basis of a Ke discount rate consistent from year to year with the project D/E from year to year, which, in turn, is consistent with the years following thereafter.

Line 121 also indicates the total project value or the total enterprise value (TEP). At the beginning of every year this value arises from the sum of debt and equity and indicates the total value of the cash flows generated by a project or by an enterprise that will go to sponsors (i.e. the shareholders and banks).

The total enterprise value will then be used to verify consistency as to the calculation of the NPV.

9. WACC Methodology with Variable D/E

Refer to "Project ALFA" in the Spreadsheet.XLS file

The calculation method illustrated above can be simplified by a quicker method that is based upon the weighted average cost of capital (WACC) used to finance the project. The WACC of a project or of an enterprise is defined as shown below:

WACC = Kd·(1 – T)·D/(D+E) + Ke·E/(D+E)

The discounting method based on WACC consists in discounting the cash flows assuming that there is no debt and therefore that the entire project is financed with share capital, though this is not true in reality.

Obviously, in the zero-debt scenario there is no interest and therefore the entire EBIT is subject to a hypothetical taxation that does not correspond to the real one. By so doing, the Free Cash Flow is calculated, this is then discounted at a rate equal to WACC. The expression free cash flow is typical of the WACC methodology and creates confusion in that the real cash flows of the project are very different.

From the business model of project ALFA (analysed in the previous chapter), you can extract such elements as are necessary to make this calculation. As will be seen, it perfectly matches the cash to equity methodology.

123	**WACC methodology with variable D/E year by year**							
124								**Liquidation**
125	**ELEMENTS OF THE VALUATION**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
126								
127	EBIT			0,0	200,0	100,0	100,0	
128	Taxes on EBIT			0,0	-100,0	-50,0	-50,0	
129	Depreciation			0,0	150,0	150,0	150,0	
130	Investments			-350,0	-50,0	-50,0	0,0	
131	Increase in working capital			-40,0	-60,0	-50,0	0,0	150,0
132		Free Cash Flow		-390,0	140,0	100,0	200,0	150,0
133								
134	**Determination of WACC year by year**							
135								
136	D/(D+E)			0,00	0,50	0,52	0,47	
137	E/(D+E)			1,00	0,50	0,48	0,53	
138								
139	WACC each period			11,0%	9,0%	8,9%	9,1%	
140								
141	WACC compounded		1,000	1,110	1,210	1,318	1,438	
142	Discount factor		1,000	0,901	0,826	0,759	0,695	0,695
143								
144	Discounted FCF to 31/12/2000			-351	116	76	139	104
145	PV of discounted FCF		83,5					
146	PV of the Equity = NPV		**83,5**					
147								
148	Verification of the Equity value in each period:							
149	Total enterprise value at beginning of period			83,5	482,7	386,2	320,7	
150	Debt at beginning of period			0,0	240,0	200,0	150,0	
151	Equity at beginning of period			83,5	242,7	186,2	170,7	
152	*check with Equity from cash to equity at variable D/E*			*0,00000*	*0,00000*	*0,00000*	*0,00000*	

Lines 127-131 include the elements required to calculate the FCF of each year.

In lines 136 and 137 the weight of equity and debt is calculated, year by year, using the D/E ratio determined on line 115 of the cash to equity model of the previous chapter.

In line 139 the WACC is calculated for each period using debt, Kd and Ke as calculated year by year using the cash to equity method on line 111 of the previous chapter.

The WACC so determined is used to calculate the discount factor on line 142.

The discount factor is then used to discount the Free Cash Flow of each year. Line 144.

The sum of the Free Cash Flows is the NPV of the project which is exactly the same as with the cash to equity method, i.e. 83.5.

As can be verified, the two results are identical, to whatever decimal point we make the comparison. Should it be otherwise, it would mean that there are mistakes in the calculation.

To further verify the consistency of the WACC methodology with the cash to equity methodology, you can back-calculate the equity value at the beginning of each period starting from the Total Enterprise Value and subtracting therefrom the debt at the beginning of the period. This calculation is developed in lines 149-151 and the equity value obtained is the same as in the previous chapter under line 113.

Once the cash to equity and the WACC valuations have been developed, the analyst can verify in a number of ways the consistency of the two valuation systems. Below is an example of this:

The NPV of the project is 83.5, which represents the value for shareholders of "owning" the project though it has not yet been implemented. By not selling the project to a third party, the shareholders waive "ready cash" of 83.5.

The equity value at the beginning of 2003 was determined to be 186.2 at line 113 of the previous chapter, column 2003.

Is it correct for equity to shift from 83.5 to 186.2 in two years? Let us verify this. The equity at the beginning of 2003 can be reconstructed starting from the beginning of the project:

83.5 x (1+0.11) = 92.7 = Equity at the end of 2001 as revalued with the 2001 Ke

92.7 + 150 = 242.7 = Equity at the end of 2001 after capital pay-in

242.7 x (1+0.14) = 276.6 = Equity at the end of 2002 as revalued with the 2002 Ke

276.7 – 90.4 = 186.2 Equity at the end of 2002 after dividend distribution

Equity at the end of 2002 after dividend distribution is the same as the equity at the beginning of 2003, i.e. 186.2.

Debt at the beginning of 2003 is 200.0, hence the total enterprise value at the beginning of 2003 is 386.2.

Is it correct that the total enterprise value as calculated with WACC shifts from 83.5 to 386.2 in two years? Let us verify this, here again, starting from year 1.

83.5 x (1+0.11) = 92.7 = TEP value at the end of 2001 as revalued with the 2001 WACC

92.7 + 390 = 482.7 = TEP value at the end of 2001 after addition of investments and working capital at the end of 2001

482.7 x (1+ 0.09) = 526.2 = TEP at the end of 2002 as revalued with the 2002 WACC

526.2 – 140 = 386.2 = TEP at the end of 2002 after distribution of the Free Cash Flow

The total enterprise value at the end of 2002 after distribution of the free cash flow is the total enterprise value at the beginning of 2003.

The same can be done with debt, as shown in chapter 7.

The value of equity, of debt and the total enterprise value must be capable of being verified in any period: either starting from the first period and revaluating the respective cash flows from time to time as a function of Ke and WACC and Kd, or starting from the last period and discounting back the cash flows with the respective Ke, WACC and Kd.

This consistency can and need be verified for any one period in the life of a project: otherwise it means that there are mistakes in the calculation.

10. Valuation Methodology with Constant D/E

Refer to "Constant D/E ratio" in the Spreadsheet.XLS file

In the previous chapters we have verified that the cash to equity methodology and the WACC methodology are equivalent and lead to the same NPV. We have defined them based on a variable D/E ratio in that the debt to equity ratio varies from year to year.

As mentioned earlier, the D/E ratio is defined as the ratio between the market value of debt and the market value of equity, where the notion of market value is similar to the value of the discounted cash flows to shareholders.

In everyday practice, i.e. when discussing investment with potential lenders or inside the organisation to which it is submitted for approval, you are faced with the need to make the project D/E explicit, which is difficult when the D/E changes from year to year as part of a broad range of values.

In the academic literature this problem is overcome assuming a constant D/E and applying the WACC methodology, avoiding, however, to make explicit the consequences on the balance sheet and on the cash flow statement. Indeed, many ignore the real meaning of a valuation with constant D/E ratio.

Let us go back to project ALFA and assume that you want to implement it with a D/E of 1.5

The project input data is the same as in the two previous chapters and allows to determine the Free Cash Flow of each year. Let us now apply the WACC valuation methodology.

Since D/E = 1.5, Ke = 15.5%, line 111, is be used throughout the life of the project:

102	**Determination of Ke valid for each year**	**2000**	**2001**	**2002**	**2003**	**2004**
103						
104	Kd		8%	8%	8%	8%
105	Rp		6%	6%	6%	6%
106	Rf		5%	5%	5%	5%
107	Taxes		50%	50%	50%	50%
108	F		1,75	1,75	1,75	1,75
109	Beta assets		1,00	1,00	1,00	1,00
110	Beta equity		1,75	1,75	1,75	1,75
111	Ke		15,5%	15,5%	15,5%	15,5%

Likewise, WACC is determined to be 8.6%, line 139:

134	**Determination of WACC valid for each year**	**2001**	**2002**	**2003**	**2004**
135					
136	D/(D+E)	0,60	0,60	0,60	0,60
137	E/(D+E)	0,40	0,40	0,40	0,40
138					
139	WACC	8,6%	8,6%	8,6%	8,6%

In the next following spreadsheet, the calculation of NPV is developed, starting from the Free Cash Flow of Project ALFA, lines 127…132.

The Free Cash Flow is discounted using WACC in lines 141…145 and shows an NPV of 89.3

123	**WACC methodology with constant D/E = 1,5**						
124							**Liquidation**
125	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
126							
127	EBIT		0,0	200,0	100,0	100,0	
128	Taxes on EBIT		0,0	-100,0	-50,0	-50,0	
129	Depreciation		0,0	150,0	150,0	150,0	
130	Investments		-350,0	-50,0	-50,0	0,0	
131	Increase in working capital		-40,0	-60,0	-50,0	0,0	150,0
132	Free Cash Flow		-390,0	140,0	100,0	200,0	150,0
133							
134	**Determination of WACC valid for each year**		**2001**	**2002**	**2003**	**2004**	
135							
136	D/(D+E)		0,60	0,60	0,60	0,60	
137	E/(D+E)		0,40	0,40	0,40	0,40	
138							
139	WACC		8,6%	8,6%	8,6%	8,6%	
140							
141	Compounded WACC	1,000	1,086	1,179	1,281	1,391	
142	Discount factor	1,000	0,921	0,848	0,781	0,719	0,719
143							
144	Discounted FCF to 31/12/2000		-359,1	118,7	78,1	143,8	107,8
145	PV of discounted FCF	89,3					
146	PV of the Equity = NPV	**89,3**					
147							
148	Verification of the Equity value in each period:						
149	Total enterprise value at beginning of period	89,3	89,3	487,0	388,8	322,3	
150	Debt at beginning of period	0,0	53,6	292,2	233,3	193,4	
151	Equity at beginning of period	89,3	35,7	194,8	155,5	128,9	
152	*check with Equity from cash to equity at D/E=1*	*0,00000*	*0,00000*	*0,00000*	*0,00000*	*0,00000*	

Therefore, if project ALFA were financed throughout its life with a constant D/E of 1.5, it would generate for shareholders an NPV of 89.3

The WACC methodology with constant D/E has been extensively used for decades in that it is easy to apply. At the end of the day, it requires to develop the P&L up to the EBIT line and only uses a simplified balance sheet based on the investments and the working capital. Most of the users of the WACC methodology with constant D/E ignore or neglect the consistency of this methodology with all other items of the P&L, the balance sheet and the cash flow statement. This, at times, leads to valuation mistakes.

What does it really mean to maintain a constant D/E of 1.5 throughout the life of the project?

Which would be the implications for the P&L and for the balance sheet if we really wanted to manage the payouts to shareholders in such a way that the D/E is 1.5 year by year?

In the cash to equity methodology with variable D/E, given a series of cash flows to shareholders, one determines the Ke for each period, which, as used to discount cash flows, allowed to determine an Equity value consistent with the D/E for the period.

If, instead, you want to maintain year by year a pre-determined and constant D/E, and hence a pre-determined and constant Ke, you need to determine the cash flow to shareholders year by year to the effect that the equity, as calculated by discounting the cash flows with Ke, is consistent with the target D/E.

The unknown factor to be determined is the distribution of cash to shareholders year by year. In contrast with the variable D/E methodology, which assumes that dividends are distributed year by year to the extent of net earnings, in order to build a constant D/E model you need to assume that cash is distributed to shareholders depending on the need to maintain the D/E constant irrespectively of the net earnings for the year. Cash distributions to shareholders are subject to maintaining the target D/E and cannot be decided *a priori*.

In any one period in the life of a project, we have the following equations:

(1) $(D)_{n+1} = (D)_n - EAT - EC - Amm + (Div)_n + Awc + Inv$

where:

$(D)_{n+1}$	**Debt at the beginning of period *n+1* and constant throughout period *n+1***
$(D)_n$	**Debt at the beginning of period *n* and constant throughout period *n***
$(Div)_n$	**Distributions of cash to shareholders at the end o period *n***
EAT	**EAT at the end of period *n***
EC	**Capital increases by shareholders at the end of period *n***
Amm	**Amortisation and depreciation in period *n***
Awc	**Increases in working capital materialising at the end of period *n***
Inv	**Investments materialising at the end of period *n***

(2) $EAT = (EBIT - Kd \cdot (D)_n) \cdot (1 - T)$

where:

EBIT	**operating profit of period *n***
Kd	**Cost of money lent by banks**
T	**Taxes**

(3) $(E)_n = ((E)_{n+1} + (Div)_n) / (1 + Ke)$

where:

$(E)_n$	**Equity value calculated at the beginning of period *n***
$(E)_{n+1}$	**Equity value calculated at the beginning of period *n+1***
Ke	**Cost of Equity**

(4) $(D/E)_n = (D/E)_t = 1.5$

where:

$(D/E)_n$	**D/E ratio of period *n***
$(D/E)_t$	**target D/E ratio ("t" for target)**

By solving (1), (2), (3) and (4) as a function of $(Div)_n$ one obtains (5):

$$(5)\quad (Div)_n = \frac{((1+Ke)\cdot(EBIT\cdot(1-T)+Amm+EC-Awc-Inv+(D)_{n+1})-(D/E)_t\cdot(E)_{n+1})\cdot(Kd\cdot(1-T))}{(1+(D/E)_t+Kd\cdot(1-T)\cdot(D/E)_t+Ke)}$$

Let us go back to the data of project ALFA and apply the formula to the line of dividends and distributions to shareholders. Line 81 of the cash flow statement:

70	**CASH FLOW STATEMENT**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
71							
72	Sources of funds						
73	EAT		-2,1	88,3	40,7	42,3	0,0
74	Depreciation		0,0	150,0	150,0	150,0	0,0
75	Equity contribution	0,0	0,0	0,0	0,0	0,0	0,0
76	**Total sources**	0,0	-2,1	238,3	190,7	192,3	0,0
77							
78	Uses of funds						
79	Investments		350,0	50,0	50,0	0,0	0,0
80	Increase in working capital		40,0	60,0	50,0	0,0	-150,0
81	Dividends and capital distributions	53,6	-153,5	69,4	50,7	88,9	60,0
82	**Total uses**	53,6	236,5	179,4	150,7	88,9	-90,0
83							
84	**Sources minus uses of funds**	-53,6	-238,6	58,9	39,9	103,4	90,0
85							
86	Net cash position beginning of period	0,0	-53,6	-292,2	-233,3	-193,4	-90,0
87	Sources minus uses of funds	-53,6	-238,6	58,9	39,9	103,4	90,0
88	Net cash ending of period	-53,6	-292,2	-233,3	-193,4	-90,0	0,0
89							
90	**Cash to Equity methodology with constant D/E = 1,5**						
91							**Liquidation**
92	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
93							
94	Debt at liquidation						90,0
95	Equity at liquidation						60,0
96	Total enterprise value at liquidation (D+E)						150,0
97	Debt at beginning of period	0,0	53,6	292,2	233,3	193,4	90,0
98	Cash to shareholders at the end of period:						
99	(dividends + capital distributions - equity contribution)	53,6	-153,5	69,4	50,7	88,9	60,0
100							
101							
102	**Determination of Ke valid for each year**	**2000**	**2001**	**2002**	**2003**	**2004**	
103							
104	Kd		8%	8%	8%	8%	
105	Rp		6%	6%	6%	6%	
106	Rf		5%	5%	5%	5%	
107	Taxes		50%	50%	50%	50%	
108	F		1,75	1,75	1,75	1,75	
109	Beta assets		1,00	1,00	1,00	1,00	
110	Beta equity		1,75	1,75	1,75	1,75	
111	Ke		15,5%	15,5%	15,5%	15,5%	
112							
113	PV of Equity at the beginning of each period = NPV	**89,3**	35,7	194,8	155,5	128,9	
114							
115	D/E of each period		1,500	1,500	1,500	1,500	
116							
117	Note: the value of the equity at the beginning of one period is calculated by discounting one						
118	year the cash flow to shareholders at the end of the period together with the value of the						
119	equity calculated for the following period with the same methodology						
120							
121	Total enterprise value (D+E) at beginning of the period	89,3	89,3	487,0	388,8	322,3	

Always start from the last period and trace back the cash flow statement taking account of the distribution of cash to shareholders as calculated with (5) which determines

(Div)*n* for each period. Obviously, the cash flow statement will also produce the net financial position at the end of the period, which will be used to complete the balance sheet. In turn, the balance sheet will determine the P&L in that the interest paid in the period will be calculated on the basis of the opening financial position.

Or, to be more precise, let us do it step by step:

As at 1.1.2005 the business is liquidated for a value of 150. Since the (D/E)*t* ratio is 1.5, clearly the proceeds from liquidation will have to be used to repay 90 (i.e. the residual debt) and 60 will go to shareholders, which is therefore the liquidation equity. Lines 94 and 95.

Therefore we have determined (D)*n+1* where *n* is 2004 and (D)*n+1* is the remaining debt as at 1.1.2005 that would apply throughout 2005 in the absence of liquidation.

In order to calculate (E)*n+1* you need to bear in mind that the distribution of 60 to shareholders upon liquidation at 1.1.2005 is not the only component of (E)*n+1;* you also need to determine the cash distribution at 31.12.2004.

In line 81 we apply (5) and find that at 31.12.2004 you need to provide for the distribution of 88.9 to shareholders. Over and above this, you have to add the 60 distributed on 1.1.2005 to liquidate the business.

Knowing that Ke = 15.5%, we can calculate (E)*n*, the equity applicable for 2004.

(E)*n* = (60 + 88.9)/(1+0.155) = 128.9 line 113, column 2004.

In line 97, knowing that (D/E)*t* is 1.5, you calculate (D)*n*, i.e. debt at the beginning of 2004 that remains constant throughout the year until 31.12.2004.

(D)*n* = 1.5 * 128.9 = 193.4

Starting from (D)*n* for 2004, we can calculate the interest to be charged to the P&L for 2004:

Interest = 193.4 * 0.08 = 15.5 line 48 column 2004

The P&L thus generates an EAT of 42.3, as indicated in line 51 column 2004.

Let us now verify if our accounts agree with the cash flow statement. Lines 72…88

The EAT is 42.3 and amortisation is 150, hence cash generation is 192.3.
Cash distribution to shareholders at the end of the period is 88.9

The difference of 103.4 is positive, i.e. the sources of cash exceed the uses thereof.

Therefore, if debt at the beginning of 2005 is 90, it means that it has gone down by 103.4 compared to the beginning of 2004.

Therefore, debt at the beginning of 2004 was 90.0+103.4 = 193.4, as indicated on line 86.

Line 86 is the same as line 97, where Debt for 2004 has been calculated to be 193.4 based on (D/E)*t* = 1.5

The system is consistent, calculations do triangulate.

If you go through the spreadsheet, you can verify the complete development of the calculation of equity and debt, period by period.

For 2003:

(Div)*n* = 50.7 distributed to shareholders on 31.12.2003
(D)*n* = 233.3 constant from 1.1.2003 to 31.12.2003
(E)*n* = 155.5 Equity value at 1.1.2003 obtained by discounting (Div)*n*+(E)*n+1*

For 2002:

(Div)*n* = 69.4 distributed to shareholders on 31.12.2002
(D)*n* = 292.2 constant from 1.1.2002 to 31.12.2002
(E)*n* = 194.8 Equity value at 1.1.2002 obtained by discounting (Div)*n*+(E)*n+1*

For 2001:

(Div)*n* = -153.5 paid by shareholders on 31.12.2001
(D)*n* = 53.6 constant from 1.1.2001 to 31.12.2001
(E)*n* = 35.7 Equity value at 1.1.2001 obtained by discounting (Div)*n*+(E)*n+1*

For 2000:

(Div)*n* = 53.6 distributed to shareholders on 31.12.2000

Let us dwell on the year 2001 for some remarks. Debt at the beginning of 2001, as determined by the model, is 53.6.

This means that at the time the decision is made, at the end of 2000, when no investment has been made as yet, the project must have already collected 53.6 of debt from banks. Furthermore, this money is distributed to shareholders as indicated in line 81, column 2000 of the cash flow statement.

In practice, if you want to operate with a constant D/E of 1.5 throughout the life of the project, from decision to liquidation, the project company must withdraw a loan of 53.6 to immediately distribute it to its shareholders since inception. In line 65, column 2000 of the balance sheet it can be seen that the project company starts off with a negative share capital of 53.6, at book value.

The total value that shareholders receive is 53.6, distributed at the time the decision is made, plus 35.7 NPV of cash distributions over the coming years as discounted with Ke. The total, 89.3, is exactly the same value as determined using the WACC methodology with constant D/E ratio. Furthermore, the ratio between the initial debt of 53.6 and the equity value as discounted at 31.12.2000, i.e. 35.7, is identical to (D/E)*t* = 1.5

The cash to equity methodology with constant D/E ratio is consistent with the WACC methodology with constant D/E ratio and leads to the same result.

What is the financial meaning of the initial debt of 53.6?
How can you interpret this apparently strange outcome? Even assuming that we found a bank that finances the project that way and assuming also that the project company is capable of legally distributing to shareholders the cash freshly collected from banks,

This is the essence of the entire financial theory of Modigliani-Miller. To find an answer, the reader is invited to read the well-known finance text *Principles of Corporate Finance, by Richard Brealey and Stewart Myers. 1984 Edition, ISBN 0-07-Y66202-9.* In particular, I refer to Chapter *19 – Interactions of investment and financing decisions*.

Ultimately, a project with a positive NPV has the effect of increasing the debt capacity of the enterprise that implements it.

The increase in debt capacity is:

$NPV \cdot D/(D+E) = NPV \cdot (D/E)t/(1+(D/E)t)$ where $(D/E)t = D/E\ target$

This is intuitive. Take the example of project ALFA. The project company is completely empty at the time it is created and at the time it is about to implement the project. The future cash flows have an NPV of 89.3.
This means that, net of the investments, the company, which, for the time being is empty, already has a market equity value of 89.3 and a debt of zero.

Naturally, aiming at a certain target D/E, debt and equity need be adjusted from the beginning, i.e. from time zero when the project takes shape. Otherwise, it would be impossible to create the pre-requisites for a constant D/E throughout the project life.

Hence, the adjustment materialises with the distribution of cash to shareholders against debt, in such a way that the D/E ratio attains the target value. In our case, the shareholders of the project company have an NPV of 89.3; they receive 53.6 thereof at time zero, and at the same time the project company takes 53.6 of debt. The difference between 89.3 and 53.6, i.e. 35.7, is the discounted value of future distributions and/or cash payments by shareholder to the enterprise. The ratio between the 53.6 debt at time zero and 35.7, i.e. the value of equity at time zero, is the target D/E of 1.5.

This same target D/E is maintained year by year and is verified at the time of liquidation. All of this means having a constant D/E throughout the life of the project.

In the example given in this chapter, the target D/E of 1.5 has been chosen for the sake of illustration only. The choice of the optimal D/E for a project or for an enterprise is not the objective of this manual; for further details, read the various specialist publications on this topic.

It is clear to the reader that no project will ever be implemented with a strictly constant D/E. The problem for the analyst is that normally the P&L, the balance sheet and the cash flow statement of a project are not structured to show a constant D/E; rather, they result from the dividend distribution policies and from the covenants imposed by the banks as a condition to grant loans.

Suffice it to compare the original balance sheet and P&L of project ALFA in chapter 8 with those of the model with D/E = 1.5 built above to see that they are different. Line

48 (interest) and line 51 (EAT in the P&L) are different. While in the balance sheet, debt (line 64) and net equity (line 65) are different.

Below is the development of the P&L and of the balance sheet of the model with constant D/E = 1.5.

41	**P&L**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	
42							
43	Revenues		0,0	600,0	1.000,0	1.000,0	
44	Costs		0,0	-250,0	-750,0	-750,0	
45	EBITDA		0,0	350,0	250,0	250,0	
46	Depreciation		0,0	-150,0	-150,0	-150,0	
47	EBIT		0,0	200,0	100,0	100,0	
48	Interests		-4,3	-23,4	-18,7	-15,5	
49	EBT		-4,3	176,6	81,3	84,5	
50	Taxes		2,1	-88,3	-40,7	-42,3	
51	EAT		-2,1	88,3	40,7	42,3	
52							
53							
54	**BALANCE SHEET**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
55							
56	Gross fixed assets	0,0	350,0	400,0	450,0	450,0	450,0
57	Cumulated depreciation	0,0	0,0	-150,0	-300,0	-450,0	-450,0
58	Net Fixed Assets	0,0	350,0	250,0	150,0	0,0	0,0
59							
60	Working capital	0,0	40,0	100,0	150,0	150,0	0,0
61							
62	TOTAL ASSETS	0,0	390,0	350,0	300,0	150,0	0,0
63							
64	Debt	53,6	292,2	233,3	193,4	90,0	0,0
65	Share capital	-53,6	97,8	116,7	106,6	60,0	0,0
66	TOTAL LIABILITIES	0,0	390,0	350,0	300,0	150,0	0,0
67							
68	*check*	*0,000*	*0,000*	*0,000*	*0,000*	*0,000*	*0,000*

11. The D/E Representative of a Project

Refer to "Project ALFA" in the Spreadsheet.XLS file

We have seen that, unless we make explicit all of the effects arising from the scenario with constant D/E, and hence, unless we deeply modify the cash flow statement, the balance sheet and the P&L, no representation of an investment is build to work with a constant D/E.

Let us go back to project ALFA and let us see, line 115, that the D/E during its life changes significantly, shifting from zero in year 0 up to a maximum of 1.07 in 2003 only to go down to zero again at the time of liquidation:

						Liquidation
102	**Determination of D/E, F and Ke year by year**	**2001**	**2002**	**2003**	**2004**	**2005**
104	Kd	8%	8%	8%	8%	
105	Rp	6%	6%	6%	6%	
106	Rf	5%	5%	5%	5%	
107	Taxes	50%	50%	50%	50%	
108	F	1,00	1,49	1,54	1,44	
109	Beta assets	1,00	1,00	1,00	1,00	
110	Beta equity	1,00	1,49	1,54	1,44	
111	Ke	11,0%	14,0%	14,2%	13,6%	
113	PV of Equity at the beginning of each period	**83,5**	242,7	186,2	170,7	
115	D/E of each period	0,00	0,99	1,07	0,88	0,00

The question is whether there can exist an average D/E of the project. Assuming there was one, how could you determine it?

There is no such a thing as an average D/E according to the definition of the term "mean". Indeed, it is not possible to determine the arithmetic mean of D/E or any other kind of mean calculation (geometrical, weighed and logarithmic) that can be used throughout the life of the project and resulting in the same NPV.

It is, however, possible to calculate a "representative" D/E of the project that allows to obtain an NPV very close to the exact NPV as calculated using the cash to equity methodology with variable D/E.

The calculation methodology rests upon the consideration that the NPV obtained using the cash to equity methodology must be the same as the NPV obtained using the WACC methodology. Therefore, if there exists a D/E that allows to define a Ke and a WACC, which, in turn, allow to calculate the same value of NPV, such D/E is the representative D/E of the project.

Let us take the model of project ALFA and assume to discount the flows to shareholders according to a Ke derived from a given D/E. Let us do the same with the Free Cash Flow that we will discount with a WACC derived from the D/E used for the cash to equity.

With a number of iterations, we will find the D/E that makes the NPVs calculated with the two methods identical. The D/E thus determined is the best possible representation of the D/E which, constantly applied throughout the life of the project, allows us to valuate its NPV.

This is referred to as the triangulation method.

The NPV determined with the triangulation methodology comes very close to the NPV calculated with the cash to equity methodology. Based upon my experience, I have never found a difference exceeding few percentage points.

The triangulation methodology is as follows:

155	**Determination of the D/E representative of the project**							
156								
157	**ELEMENTS OF THE VALUATION**							
158		Kd	8%					
159		Rf	6%					
160		Rp	5%					
161		Taxes	50%					
162		F	1,42					
163		Beta Assets	1,00					
164		Beta Equity	1,42					
165		Ke	13,5%	representative of the project				
166		D/E	**0,84**	calculated with reiterations until the value of equity is the same				
167		D/(D+E)	0,46					
168		E/(D+E)	0,54					
169		WACC	9,2%	representative of the project				
170								**Liquidation**
171	**CASH TO EQUITY**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
172								
173	Cash to shareholders:			-150,0	90,4	42,0	44,0	150,0
174	Compounded Ke		1,000	1,135	1,289	1,463	1,661	
175	Discount factor		1,000	0,881	0,776	0,683	0,602	0,602
176	discounted cash to shareholders			-132,1	70,1	28,7	26,5	90,3
177								
178		Equity value on 31/12/2000	**83,5**					
179								
180								**Liquidation**
181	**WACC**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
182								
183	Free Cash Flows			-390,0	140,0	100,0	200,0	150,0
184	Compounded WACC		1,000	1,092	1,192	1,301	1,420	
185	Discount factor		1,000	0,916	0,839	0,769	0,704	0,704
186	Discounted FCF			-357,2	117,5	76,9	140,8	105,6
187								
188		Enterprise value on 31/12/2000	83,5					
189		Debt on 31/12/2000	0,0					
190		Equity value on 31/12/2000	**83,5**					
191								
192								
193		check of the reiteration	1,000	Reiterate D/E until the ratio between the equity value from Cash to Equity and				
194				from WACC have the same value				

Lines 173 and 183 show such cash flows of project ALFA as are required to calculate the NPV either with the Cash To Equity or with the WACC methodology.

In order to make the calculation you have to set the cell of line 166 in such a way that the two NPVs calculated with the Cash To Equity methodology, line 178, and with the WACC methodology, line 190, are identical. This occurs when the ratio between the two NPVs is exactly equal to 1, line 193. The calculation is made using the Excel target

function. Set the target value to 1 for the cell of line 193 by changing the cell of line 166. The iterative calculation must at times be repeated 2 or 3 times for the purposes of convergence.

The two NPVs are the same where D/E = 0.84, which entails a Ke of 13.5% and a WACC of 9.2%.

Both the cash to equity methodology, line 178, and the WACC methodology, line 190, show the same NPV of 83.5.

This is the same value that we had obtained in chapters 8 and 9. Furthermore, now we have a D/E representative of the project and the corresponding Ke and WACC are consistent with the results obtained.

Please note that the NPVs obtained with triangulation and the NPV obtained in chapters 8 and 9 are not exactly the same.

To be precise, the NPV of chapters 8 and 9 is 83.494 whereas the NPV of this chapter is 83.544. This difference is inevitable, considering that the triangulation methodology is an empirical one. Like I said earlier, however, I have never found differences exceeding some percentage points.

The triangulation methodology offers a number of advantages:

a. **It is quick in that it does not need to calculate the D/E for every period and avoids very complicated formulas to be entered in the cells of the model.**
b. **It allows to determine an NPV that is identical or very similar to the one that can be obtained by strictly applying the more classic discounting methods.**
c. **It allows to determine a D/E representative of the project and a corresponding Ke and WACC.**

This is my preferred method, which I have applied to many valuations of projects and enterprises.

12. The EVA Methodology

Refer to "Project ALFA" in the Spreadsheet.XLS file

This manual presents an overview of the various systems of discounting techniques to determine the value of a project or of an enterprise. As part of this, mention should be made of Economic Value Added, EVA.

EVA is a way of "representing" the calculations that have already been made in the previous chapters, without altering the result.

EVA is defined as the incremental return that shareholders receive from a given investment as compared to the expected return as calculated on the book value of the investment.

Application of EVA reflects the following rationale:

At the beginning of each period the shareholders assign to the management a net invested capital to be managed so as to derive profit from the same.
The expected return for the year is equal to the WACC applied to the net invested capital.
The actual return attained by the management is referred to as NOPAT (net operating profit after tax).
The difference between actual return and expected return is EVA.
Where EVA is higher than zero, this means that the management has created value, i.e. that it has turned the net invested capital to profit over and above the expectations of shareholders.

EVA has become widespread in relation to management remuneration and bonus systems in that it is easy to apply and understand for all participants, be they shareholders or managers.

Calculation of EVA assumes the existence of a reference WACC, which is why I deal with it after explaining the triangulation methodology which allows to determine the representative D/E and the corresponding WACC.

EVA can be determined on the Net Invested Capital (NIC) or on the Gross Invested Capital (GIC) at book values.

EVA on Net Invested Capital

For project ALFA, the elements required to determine it are the following:

Line 259. Net Invested Capital at the beginning of each period, or Net Fixed Assets + Working Capital taken directly from the balance sheet of the project.

Line 260. NOPAT (net operating profit after tax) expected from that type of project. It is calculated as WACC multiplied by NIC.

Line 261. Actual NOPAT, i.e. the fully taxed EBIT of each period.

Line 262. EVA is calculated as the difference between expected NOPAT and actual NOPAT.

Please note that EVA is present also at liquidation. Obviously, no expected rate is applied in that liquidation is an event that occurs at a given point in time.
With project ALFA, the shareholders give the management a NIC of 150 as at 1.1.2005.
The management sells all for 150 as at 1.1.2005.
Therefore, EVA at liquidation is 150-150 = 0.

In lines 264 and 265 EVA is discounted using WACC.

255	**Determination of EVA on NIC using WACC =**	**9,2%**	From the triangulation method					
256								
257								**Liquidation**
258			**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
259	NIC beginning of the period			0,0	390,0	350,0	300,0	
260	expected NOPAT			0,0	35,8	32,1	27,5	
261	achieved NOPAT = EBIT*(1-T)			0,0	100,0	50,0	50,0	
262	EVA			0,0	64,2	17,9	22,5	0,0
263								
264	Discount factor based on WACC		1	0,916	0,839	0,769	0,704	0,704
265	Discounted EVA			0,0	53,9	13,8	15,8	0,0
266								
267	Present Value of EVA		83,5					
268	NIC on 31/12/2000		0,0					
269	Debt on 31/12/2000		0,0					
270	NPV of the project		**83,5**					

As can be seen, the project's NPV is 83.5, as determined under chapters 8, 9 and 11.

EVA on Gross Invested Capital

The method above is used here again, though using the invested capital gross of amortisation & depreciation.

As to project ALFA, the elements required to determine it are:

272	**Determination of EVA on GIC using WACC =**	**9,2%**	from the triangulation method					
273								
274								**Liquidation**
275			**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
276	GIC beginning of period			0,0	390,0	500,0	600,0	
277	expected NOPAT			0,0	35,8	45,9	55,0	
278	achieved NOPAT + Depreciation			0,0	250,0	200,0	200,0	
279	EVA			0,0	214,2	154,1	145,0	-450,0
280								
281	Discount factor based on WACC		1	0,916	0,839	0,769	0,704	0,704
282	Discounted EVA			0,0	179,8	118,5	102,1	-316,8
283								
284	Present Value of EVA		83,5					
285	GIC on 31/12/2000		0,0					
286	Debt on 31/12/2000		0,0					
287	NPV of the project		**83,5**					
288								

Line 276. Gross Invested Capital at the beginning of each period, or Gross Fixed Assets + Working Capital taken directly from the balance sheet of the project.

Line 277. Expected NOPAT, calculated as WACC multiplied by GIC.

Line 278. Actual NOPAT, or the fully taxed EBIT of each period added to the amortisation shown in the P&L.

Line 279. EVA is calculated as the difference between expected NOPAT and actual NOPAT.

Please note that EVA on GIC is broadly negative for project ALFA at liquidation.

The shareholders give the management a GIC of 600 as at 1.1.2005 comprised of gross investments of 450 and operating capital of 150. The management sells all for 150 as at 1.1.2005.

Hence, EVA at liquidation is 150-600 = - 450.

In lines 281 and 282 the EVA is discounted using WACC.

Here again, the NPV of the project is 83.5, i.e. exactly the same as in chapters 8, 9 and 11 and as determined with EVA on NIC.

The use of Gross Invested Capital as reference basis for calculating EVA may appear strange but it is, indeed, used by some enterprises, most especially by very old ones where the basis of assets has been fully amortised or where some assets have been written off. Enterprises like these opt for the use of EVA on GIC in that the NIC has "lost memory" of all the investments made in the past.

We have, however, demonstrated that the two methods are equivalent in terms of discounting maths; the choice of one or the other rests with the enterprise and the shareholders and lies beyond the scope of this manual.

Having come thus far, with a given project, like for example project ALFA, the analyst has a number of mutually consistent tools to determine the value of a project.

The table below summarises the NPVs of project ALFA with all of the methods described.

29	**OUTPUT ZONE**	
30		
31	**Equity value on 31/12/2000 using D/E at proxy market value**	**NPV**
32	Cash to equity with variable D/E	**83,5**
33	WACC with variable D/E	**83,5**
34	Triangulation	**83,5**
35	EVA on Net Invested Capital	**83,5**
36	EVA on Gross Invested Capital	**83,5**

There is no denying that this is useful for the valuator. NPV is determined with 5 different methods that show converging results, lines 32…34.

Furthermore, the D/E representative of the project, and the corresponding WACC and Ke have been determined.

The same NPV can be represented with two systems for defining the EVA, which, in turn, can be used as an incentive for the management.

13. Valuation Methods Using Book Value D/E

Refer to "Project ALFA" in Spreadsheet.XLS file

We have shown that using D/E determined on a discounted cash flow basis, to replace market-value D/E, allows to refer the calculation of NPV to a number of mutually consistent methods.

It is worthwhile spending one chapter on the consequences of using book-value D/E. It is useful to dwell upon this at least to clear any doubts on the uselessness and dangers of using D/E calculated on book values. The book values of an enterprise are most of the times totally unrelated to market values and, as a result, generate inconsistent and incorrect results.

Let us take project ALFA and calculate Ke and WACC with the D/E taken from book values. Lines 210 and 214.

196	**Determination of Ke and WACC using book values**					
197						
198			**2001**	**2002**	**2003**	**2004**
199	Debt at beginning of the period		0,0	240,0	200,0	150,0
200	Book value of equity at beginning of period		0,0	150,0	150,0	150,0
201	Total enterprise value at beginning of period		0,0	390,0	350,0	300,0
202						
203		Kd	8%	8%	8%	8%
204		Rf	5%	5%	5%	5%
205		Rp	6%	6%	6%	6%
206		Taxes	50%	50%	50%	50%
207		F	1,00	1,80	1,67	1,50
208		Beta Assets	1,00	1,00	1,00	1,00
209		Beta Equity	1,00	1,80	1,67	1,50
210		Ke at book value year by year	11,0%	15,0%	14,3%	13,5%
211		D/E book value	0,000	1,600	1,333	1,000
212		D/(D+E) book value	61,5%	61,5%	57,1%	50,0%
213		E/(D+E) book value	38,5%	38,5%	42,9%	50,0%
214	WACC at book value year by year		6,7%	8,2%	8,4%	8,8%

If you take from the cash flow statement the cash flows from/to shareholders, you can determine the NPV with the cash to equity method: the result is an NPV of 81.6, line 222.

216	**CASH TO EQUITY**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
217	**Variable Ke year by year**						
218	Cash to shareholders:		-150,0	90,4	42,0	44,0	150,0
219	Compounded Ke	1,000	1,110	1,277	1,459	1,656	
220	Discount factor	1,000	0,901	0,783	0,685	0,604	0,604
221	Discounted cash to shareholders		-135,1	70,8	28,8	26,6	90,6
222	Equity value on 31/12/2000	**81,6**					
223							

Likewise, it is possible to determine the NPV with the Free Cash Flow by using the WACC method, with a resulting NPV of 92.6. Line 233.

224	**WACC**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
225	**Variable WACC year by year**						
226	Free Cash Flow		-390,0	140,0	100,0	200,0	150,0
227	Compounded WACC	1,000	1,067	1,155	1,252	1,362	1,362
228	Discount factor	1,000	0,937	0,866	0,799	0,734	0,734
229	Discounted FCF		-365,5	121,2	79,9	146,9	110,2
230							
231	TEP on 31/12/2000	92,6					
232	Debt on 31/12/2000	0,0					
233	Equity Value on 31/12/2000	**92,6**					
234							

The first remark is that the two NPVs are not the same, while they should be. Also, by chance, the NPV determined using cash to equity is not so far from that determined in chapter 8, whereas the NPV calculated with the WACC method is totally different from that determined in chapter 9.

The question arises as to which of the two is the correct one, if any. We have no tool to determine whether or not this is the case. These are just two numbers.

I could fill whole pages with examples of how some analysts have tried, without succeeding, to use book-value D/E to attain consistent results. A typical case in point is the practice of defining the project D/E as the ratio calculated on the values of the initial investment, i.e. of year 1 of operations.

With project ALFA, in year 1 we have an investment of 350 and an increase in operating capital of 40, for a total requirement of 390. The equity paid in by shareholders is 150 and hence debt amounts to 240 and D/E = 1.6

It follows that Ke is 15.5% and WACC is 8.2%, with both remaining unaltered throughout the project.

The NPV calculated with the cash to equity method is 76.5, line 241

235	**CASH TO EQUITY**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
236	**Constant Ke equal to 1st year**						
237	Cash to shareholders:		-150,0	90,4	42,0	44,0	150,0
238	Compounded Ke	1,000	1,150	1,323	1,521	1,749	
239	Discount factor	1,000	0,870	0,756	0,658	0,572	0,572
240	Discounted cash to shareholders		-130,4	68,4	27,6	25,2	85,8
241	Equity value on 31/12/2000	**76,5**					
242							

The NPV calculated with the WACC method is 93.1, line 252

243	**WACC**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
244	**Constant WACC equal to 1st year**						
245	Free Cash Flow		-390,0	140,0	100,0	200,0	150,0
246	Compounded WACC	1,000	1,082	1,171	1,268	1,372	
247	Discount factor	1,000	0,924	0,854	0,789	0,729	0,729
248	Discounted FCF		-360,3	119,5	78,9	145,8	109,3
249							
250	TEP on 31/12/2000	93,1					
251	Debt on 31/12/2000	0,0					
252	Equity Value on 31/12/2000	**93,1**					

253

Here again there is an inconsistency between the two NPVs determined using year 1 book-value D/E as well as with the NPVs previously determined using variable D/E at book values.

The reader can take delight in trying to use book-value D/E in the most creative mixes but will never attain the same level of consistency as with D/E at proxy market-value. This can be verified by summarizing the NPVs calculated using book-value D/E and market-value D/E.

29	**OUTPUT ZONE**				
30					
31	**Equity value on 31/12/2000 using D/E at proxy market value**	**NPV**		**Equity value on 31/12/2000 using book value D/E**	**NPV**
32	Cash to equity with variable D/E	**83,5**		Cash to equity with constant D/E	**81,6**
33	WACC with variable D/E	**83,5**		WACC with constant D/E	**92,6**
34	Triangulation	**83,5**		Cash to equity with constant D/E equal to 1st year	**76,5**
35	EVA on Net Invested Capital	**83,5**		WACC with constant D/E equal to 1st year	**93,1**
36	EVA on Gross Invested Capital	**83,5**			

The diversity of results that can be obtained using book-value D/E and the inconsistency of these results can, at times, pose significant problems to the analyst.

Indeed, where the NPV of the investment is modest, it very often is the case that using book-value D/E the NPV is positive or negative depending upon whether you use the cash to equity methodology or the WACC methodology, which seriously hampers the decision-making process.

The use of D/E with proxy market values, is, instead, much more consistent. The NPV, be it calculated using cash to equity or WACC or triangulation, must at all times be the same, i.e. either positive or negative.

14. A Special Case: Variable Periods

Refer to "Variable periods" in the Spreadsheet.XLS file

In chapter 8, when project ALFA was analysed for the first time using the cash to equity methodology with a variable D/E, the life of the project was represented in 4 one-year periods, from 1.1.2001 to 31.12.2004.

An expert would immediately perceive that the description of the life of the project, especially in its initial stage, cannot be fully captured by a model developed on the basis of one-year periods. In the early stages of the project, considerable cash outflows take place, which, when discounted, have a strong impact on the NPV. A 6-month shift in a significant disbursement can change significantly the NPV.

Hence the need for a more precise allocation of cash flows over time. This can be done by defining the periods of the business model as you prefer with a view to a more precise description. For example, the model of project ALFA could be split into 8 half-year periods rather than into 4 one-year periods, or into 16 quarters. Obviously, in this case you would need to re-define Ke and Kd on a six-monthly or on a quarterly basis respectively.

If, for example, payment by banks is made before the end of year 1, as the investment progresses further, you may split year 1 into 12 months or into 4 quarters and assign to each the relevant cash movements, leaving the next following years unaltered.

Importantly, you must treat each period as a constant segment in the life of the project, leaving the variations of cash inflows and outflows at the end of each period.

Let us now develop the example shown under Variable Periods in the business model of project ALFA.

Year 2001 was broken down into 4 quarters.

10	**P&L**		31/12/2000	31/03/2001	30/06/2001	30/09/2001	31/12/2001	31/12/2002	31/12/2003	31/12/2004
11										
12	Revenues	input		0	0	0	0	600	1.000	1.000
13	Costs	input		0	0	0	0	-250	-750	-750
14	Depreciation	input		0	0	0	0	150	150	150
15										
16	**CASH FLOW**									
17										
18	Equity contribution	input		100	50	0	0	0	0	0
19	Investments	input		100	100	100	50	50	50	0
20										
21	**BALANCE SHEET**									
22										
23	Gross fixed assets	input	0	0	0	0				
24	Cumulated depreciation	input	0	0	0	0				
25	Working capital	input	0	0	0	0	40	100	150	150
26	Debt	input	0	0	0	0				
27	Share capital	input	0	0	0	0				

The share capital is paid in at the end of the first quarter and at the end of the second one. Line 18. The investment is paid to suppliers at the end of each quarter of 2001. Line 19.

The working capital is established at the end of Q4 of 2001, as provided for in the original case. The years following thereafter are the same as the previous model. The new input data is used to work out the P&L, the balance sheet and the cash flow statement.

38	**P&L**	**31/12/2000**	**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	
39										
40	Revenues		0,0	0,0	1,0	2,9	600,0	1.000,0	1.000,0	
41	Costs		0,0	0,0	0,0	0,0	-250,0	-750,0	-750,0	
42	EBITDA		0,0	0,0	1,0	2,9	350,0	250,0	250,0	
43	Depreciation		0,0	0,0	0,0	0,0	-151,3	-151,3	-151,3	
44	EBIT		0,0	0,0	1,0	2,9	198,7	98,7	98,7	
45	Interests		0,0	0,0	-1,0	-2,9	-19,5	-16,2	-12,1	
46	EBT		0,0	0,0	0,0	0,0	179,2	82,5	86,6	
47	Taxes		0,0	0,0	0,0	0,0	-89,6	-41,2	-43,3	
48	EAT		0,0	0,0	0,0	0,0	89,6	41,2	43,3	
49										
50										
51	**BALANCE SHEET**	**31/12/2000**	**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
52										
53	Gross fixed assets		100,0	200,0	301,0	353,9	403,9	453,9	453,9	453,9
54	Cumulated depreciation		0,0	0,0	0,0	0,0	-151,3	-302,6	-453,9	-453,9
55	Net Fixed Assets		100,0	200,0	301,0	353,9	252,6	151,3	0,0	0,0
56										
57	Working capital		0,0	0,0	0,0	40,0	100,0	150,0	150,0	0,0
58										
59	TOTAL ASSETS	0	100,0	200,0	301,0	393,9	352,6	301,3	150,0	0,0
60										
61	Debt		0,0	50,0	151,0	243,9	202,6	151,3	0,0	0,0
62	Share capital		100,0	150,0	150,0	150,0	150,0	150,0	150,0	0,0
63	TOTAL LIABILITIES		100,0	200,0	301,0	393,9	352,6	301,3	150,0	0,0
64										
65	*check*	0,00	0,000	0,000	0,000	0,000	0,000	0,000	0,000	0,000
66										
67	**CASH FLOW STATEMENT**	**31/12/2000**	**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
68										
69	Sources of funds									
70	EAT		0,0	0,0	0,0	0,0	89,6	41,2	43,3	0,0
71	Depreciation		0,0	0,0	0,0	0,0	151,3	151,3	151,3	0,0
72	Equity contribution		100,0	50,0	0,0	0,0	0,0	0,0	0,0	0,0
73	**Total sources**		100,0	50,0	0,0	0,0	240,9	192,5	194,6	0,0
74										
75	Uses of funds									
76	Investments		100,0	100,0	101,0	52,9	50,0	50,0	0,0	0,0
77	Increase in working capital		0,0	0,0	0,0	40,0	60,0	50,0	0,0	-150,0
78	Dividends and capital distribution		0,0	0,0	0,0	0,0	89,6	41,2	43,3	150,0
79	**Total uses**		100,0	100,0	101,0	92,9	199,6	141,2	43,3	0,0
80										
81	**Sources minus uses of funds**		0,0	-50,0	-101,0	-92,9	41,3	51,3	151,3	0,0
82										
83	Net cash position beginning of period		0,0	0,0	-50,0	-151,0	-243,9	-202,6	-151,3	0,0
84	Sources minus uses of funds		0,0	-50,0	-101,0	-92,9	41,3	51,3	151,3	0,0
85	Net cash ending of period	0	0,0	-50,0	-151,0	-243,9	-202,6	-151,3	0,0	0,0

Obviously, it has been necessary to take account of the capitalisation of the interest incurred in Q3 and Q4 and amortisation & depreciation have been modified accordingly. See: line 40 where capitalised interest is shown as revenues, line 76 where investments include such capitalisation and line 54 where amortization & depreciation have been recalculated.

The valuation according to the cash to equity methodology with variable D/E is described in below:

87	**Cash to Equity methodology with variable D/E year by year**									
88										**Liquidatio**
89	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/200**
90										
91	Debt at liquidation									0,
92	Equity at liquidation									150,
93	Total enterprise value at liquidation (D+E)									150,
94	Debt at beginning of period		0,0	0,0	50,0	151,0	243,9	202,6	151,3	
95	Cash to shareholders:									
96	(dividends + capital distributions - equity contribution)		-100,0	-50,0	0,0	0,0	89,6	41,2	43,3	150,
97										
98										
99	**Determination of D/E, F and Ke year by year**		**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	
100										
101	Kd		1,94%	1,94%	1,94%	1,94%	8%	8%	8%	
102	Rp		6%	6%	6%	6%	6%	6%	6%	
103	Rf		5%	5%	5%	5%	5%	5%	5%	
104	Taxes		50%	50%	50%	50%	50%	50%	50%	
105	F		1,00	1,00	1,11	1,32	1,51	1,55	1,44	
106	Beta assets		1,00	1,00	1,00	1,00	1,00	1,00	1,00	
107	Beta equity		1,00	1,00	1,11	1,32	1,51	1,55	1,44	
108	Ke		2,64%	2,64%	2,81%	3,13%	14,0%	14,3%	13,7%	
109										
110	PV of Equity at the beginning of each period = NPV		**70,57**	172,44	227,00	233,37	240,68	184,88	170,05	
111										
112	D/E of each period		0,00	0,00	0,22	0,65	1,01	1,10	0,89	
113										
114	Note: the value of the equity at the beginning of one period is calculated by discounting one									
115	year the cash flow to shareholders at the end of the period together with the value of the									
116	equity calculated for the following period with the same methodology									
117										
118	Total enterprise value (D+E) at beginning of the period		70,6	172,4	277,0	384,3	484,6	387,5	321,4	

Please note that the Kd in line 101 is shown on a quarterly basis for the 4 periods of 2001 and the same was done with the Ke shown in line 108 of the model.

The NPV of the project is much lower than in the previous case: 70.6 against 83.5 calculated in chapters 8 and 9.

This was to be expected. The capital contribution was made almost one year earlier and the project is slightly more expensive as a result of the capitalization of interest. However, the model now is much more precise in the description of economic factors, most especially in its initial phase.

This is the reason why in large projects, especially highly leveraged ones, financial institutions require business models which, at least for the first two years, report monthly or quarterly cash flows.

Using the WACC methodology with variable D/E year by year, the same NPV is obtained:

120	**WACC methodology with variable D/E year by year**									
121										**Liquidation**
122	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
123										
124	EBIT		0,0	0,0	1,0	2,9	198,7	98,7	98,7	
125	Taxes on EBIT		0,0	0,0	-0,5	-1,5	-99,3	-49,3	-49,3	
126	Depreciation		0,0	0,0	0,0	0,0	151,3	151,3	151,3	
127	Investments		-100,0	-100,0	-101,0	-52,9	-50,0	-50,0	0,0	
128	Increase in working capital		0,0	0,0	0,0	-40,0	-60,0	-50,0	0,0	150,0
129	Free Cash Flow		-100,0	-100,0	-100,5	-91,5	140,7	100,7	200,7	150,0
130										
131	**Determination of WACC year by year**									
132										
133	D/(D+E)		0,00	0,00	0,18	0,39	0,50	0,52	0,47	
134	E/(D+E)		1,00	1,00	0,82	0,61	0,50	0,48	0,53	
135										
136	WACC each period		2,6%	2,6%	2,5%	2,3%	9,0%	8,9%	9,1%	
137										
138	WACC compounded	1,00	1,026	1,054	1,080	1,104	1,204	1,311	1,430	
139	Discount factor	1,00	0,974	0,949	0,926	0,906	0,831	0,763	0,699	0,699
140										
141	Discounted FCF to 31/12/2000		-97,4	-94,9	-93,1	-82,8	116,9	76,8	140,3	104,9
142	PV of discounted FCF	70,6								
143	PV of the Equity = NPV	**70,6**								
144										
145	Verification of the Equity value in each period:									
146	Total enterprise value at beginning of period		70,6	172,4	277,0	384,3	484,6	387,5	321,4	
147	Debt at beginning of period		0,0	0,0	50,0	151,0	243,9	202,6	151,3	
148	Equity at beginning of period		70,6	172,4	227,0	233,4	240,7	184,9	170,1	
149	*check with Equity from cash to equity at variable D/E*		*0,00000*	*0,00000*	*0,00000*	*0,00000*	*0,00000*	*0,00000*	*0,00000*	

Let us now see the valuation of the project using the triangulation methodology shown in the table on the next following page.

NPV is 73.0 and the D/E representative of the project is 0.82.

The difference as compared to the NPV determined with cash to equity is 3%, which is an acceptable approximation.

Please note that the D/E representative of project ALFA, as calculated in chapter 11, was 0.84, i.e. very close to the D/E calculated in this chapter (0.82) and therefore not affecting the values of WACC and Ke. Still, the NPV is much lower. This is due to the fact that, with a quarterly representation, the cash disbursement to make the investment occurs much earlier and, as a result, the NPV is depressed.

Determination of the D/E representative of the project

ELEMENTS OF THE VALUATION

Kd	8%
Rf	5%
Rp	6%
Taxes	50%
F	1,41
Beta Assets	1,00
Beta Equity	1,41
Ke	13,0% representative of the project
D/E	**0,82** calculated with reiterations until the value of equity is the same
D/(D+E)	0,45
E/(D+E)	0,55
WACC	9,0% representative of the project

CASH TO EQUITY	**31/12/2000**	**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**Liquidation 1/1/2005**
Cash to shareholders:		-100,0	-50,0	0,0	0,0	89,6	41,2	43,3	150,0
Compounded Ke	1,000	1,031	1,063	1,096	1,130	1,278	1,445	1,633	
Discount factor	1,000	0,970	0,941	0,912	0,885	0,783	0,692	0,612	0,612
discounted cash to shareholders		-97,0	-47,0	0,0	0,0	70,1	28,6	26,5	91,9

PV of the Equity = NPV **73,0**

WACC	**31/12/2000**	**31/03/2001**	**30/06/2001**	**30/09/2001**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**Liquidation 1/1/2005**
Free Cash Flows		-100,0	-100,0	-100,5	-91,5	140,7	100,7	200,7	150,0
Compounded WACC	1,000	1,022	1,044	1,067	1,090	1,188	1,294	1,410	
Discount factor	1,000	0,979	0,958	0,938	0,918	0,842	0,773	0,709	0,709
Discounted FCF		-97,9	-95,8	-94,2	-83,9	118,4	77,8	142,3	106,4

TEP on 31/12/2000	73,0
Debt on 31/12/2000	0,0
PV of the Equity = NPV	**73,0**
check of the reiteration	1,000 Reiterate D/E until the ratio between the equity value from Cash to Equity and from WACC have the same value

15. A Special Case: Subsidized loan

Refer to "Subsidized loan" in the Spreadsheet.XLS file

Until now we have analysed project ALFA assuming a scenario where debt is granted at market conditions, i.e. Kd. In daily practice, however, there are some cases in which the loan is granted at rates lower than market rates or in which a part of the debt is granted as a sinking fund.

In this case, the analyst needs to build a business model which, on the one hand allows to truthfully represent what happens in the balance sheet and in the P&L under the actual debt conditions and, on the other hand, allows to correctly determine D/E and NPV.

Assume, for example, that project ALFA is financed in part with debt at a rate of 5% instead of 8%, as shown by market Kd. A subsidized loan of 150 is granted as at 31.12.2001 and is repaid in instalments of 50 each at the end of each subsequent year.

The remaining portion of the financial requirement is provided by banks at Kd at market conditions of 8%.

The input table is set out below:

1	**INPUT ZONE**						
2							
3	Kd at market conditions	Input	8,0%				
4	Kd subsidised	Input	5,0%				
5	Rp	Input	6,0%				
6	Rf	Input	5,0%				
7	Taxes	input	50,0%				
8	Beta assets	input	1,000				
9	Terminal value	input	liquidation of working capital				
10							
11	**P&L**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
12							
13	Revenues	input		0	600	1.000	1.000
14	Costs	input		0	-250	-750	-750
15	Depreciation	input		0	-150	-150	-150
16							
17	**CASH**						
18							
19	Equity contribution	input		150	0	0	0
20	Investments	input		350	50	50	0
21							
22	**BALANCE SHEET**						
23							
24	Gross fixed assets	input	0				
25	Cumulated depreciation	input	0				
26	Working capital	input	0	40	100	150	150
27	Debt	input	0				
28	Share capital	input	0				
29	Subsidized Loan	input	0	150	100	50	

The P&L, balance sheet and cash flow statement are shown below:

40	**P&L**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	
41							
42	Revenues		0,0	600,0	1.000,0	1.000,0	
43	Costs		0,0	-250,0	-750,0	-750,0	
44	EBITDA		0,0	350,0	250,0	250,0	
45	Depreciation		0,0	-150,0	-150,0	-150,0	
46	EBIT		0,0	200,0	100,0	100,0	
47	Subsidy			4,5	3,0	1,5	
48	Interests at market conditions on total debt		0,0	-19,2	-16,0	-12,0	
49	EBT		0,0	185,3	87,0	89,5	
50	Taxes		0,0	-92,7	-43,5	-44,8	
51	EAT		0,0	92,7	43,5	44,8	
52							
53	**BALANCE SHEET**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
54							
55	Gross fixed assets	0,0	350,0	400,0	450,0	450,0	450,0
56	Cumulated depreciation	0,0	0,0	-150,0	-300,0	-450,0	-450,0
57	Net Fixed Assets	0,0	350,0	250,0	150,0	0,0	0,0
58							
59	Working capital	0,0	40,0	100,0	150,0	150,0	0,0
60							
61	TOTAL ASSETS	0,0	390,0	350,0	300,0	150,0	0,0
62	Subsidized Loan	0,0	150,0	100,0	50,0	0,0	0,0
63	Debt	0,0	90,0	100,0	100,0	0,0	0,0
64	Share capital	0,0	150,0	150,0	150,0	150,0	
65	TOTAL LIABILITIES	0,0	390,0	350,0	300,0	150,0	0,0
66	*check*	*0,000*	*0,000*	*0,000*	*0,000*	*0,000*	*0,000*
67							
68	**CASH FLOW STATEMENT**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
69	Sources of funds						
70	EAT		0,0	92,7	43,5	44,8	0,0
71	Depreciation		0,0	150,0	150,0	150,0	0,0
72	Subsidized loan		150,0	-50,0	-50,0	-50,0	0,0
73	Equity contribution		150,0	0,0	0,0	0,0	0,0
74	**Total sources**		300,0	192,7	143,5	144,8	0,0
75							
76	Uses of funds						
77	Investments		350,0	50,0	50,0	0,0	0,0
78	Increase in working capital		40,0	60,0	50,0	0,0	-150,0
79	Dividends and capital distribution		0,0	92,7	43,5	44,8	150,0
80	**Total uses**		390,0	202,7	143,5	44,8	0,0
81	**Sources minus uses of funds**		-90,0	-10,0	0,0	100,0	0,0
82							
83	Net cash position beginning of period		0,0	-90,0	-100,0	-100,0	0,0
84	Sources minus uses of funds		-90,0	-10,0	0,0	100,0	0,0
85	Net cash ending of period	0,0	-90,0	-100,0	-100,0	0,0	0,0

There exist different ways of analysing a subsidized loan and, obviously, they all lead to the same result.

A subsidized loan is as if, together with a loan with Kd at market conditions, you obtained, at the same time, a subsidy directly in the P&L equal to the differential between the Kd at market conditions and the subsidized Kd. The subsidy, which, as a rule, is a government subsidy, is related to the nature of the project, the business sector or the location where the project is implemented.

This is the representation given under line 47 of the P&L, where the subsidy is made explicit and cancels one portion of the interest shown in line 48, calculated as if the entire debt were subject to Kd at market conditions.

Line 49 shows the net interest actually paid by the project enterprise.

Let us now apply the Cash To Equity methodology with variable D/E. The result is an NPV of 87.0, line 111. The NPV has increased compared to project ALFA of chapter 8 without a subsidized loan, in that a lower cost of debt can but result in a higher equity value.

88	**Cash to Equity methodology with variable D/E year by year**						
89							**Liquidation**
90	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
91							
92	Debt at liquidation						0,0
93	Equity at liquidation						150,0
94	Total enterprise value at liquidation (D+E)						150,0
95	Debt at beginning of period		0,0	240,0	200,0	150,0	0,0
96	Cash to shareholders:						
97	(dividends + capital distributions - equity contribution)		-150,0	92,7	43,5	44,8	150,0
98							
99							
100	**Determination of D/E, F and Ke year by year**		**2001**	**2002**	**2003**	**2004**	
101							
102	Kd at market conditions		8,0%	8,0%	8,0%	8,0%	
103	Rp		6%	6%	6%	6%	
104	Rf		5%	5%	5%	5%	
105	Taxes		50%	50%	50%	50%	
106	F		1,00	1,49	1,53	1,44	
107	Beta assets		1,00	1,00	1,00	1,00	
108	Beta equity		1,00	1,49	1,53	1,44	
109	Ke		11,0%	13,9%	14,2%	13,6%	
110							
111	Value of Equity at the beginning of period		**87,0**	246,5	188,2	171,4	
112							
113	D/E of each period		0,00	0,97	1,06	0,88	
114							
115	Note: the value of the equity at the beginning of one period is calculated by discounting one						
116	year the cash flow to shareholders at the end of the period together with the value of the						
117	equity calculated for the following period with the same methodology						
118							
119	Total enterprise value (D+E) at beginning of the period		87,0	486,5	388,2	321,4	

For the sake of verification, let us now also apply the WACC methodology with variable D/E. Obviously, we must remember that with WACC the entire operating profit (i.e. excluding interest expense at market values) is fully taxed.

The subsidy, as defined at the beginning of this chapter, is a contribution that a government entity pays to a given project. Hence, it must be treated as one of the operating flows of the project and, obviously, it must be subject to taxation. Line 125 shows the EBIT increased by the virtual value of the subsidy.

As was to be expected, the NPV is 87.0

121	**WACC methodology with variable D/E year by year**						
122							**Liquidation**
123	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
124							
125	EBIT + subsidy		0,0	204,5	103,0	101,5	
126	Taxes on EBIT + subsidy		0,0	-102,3	-51,5	-50,8	
127	Depreciation		0,0	150,0	150,0	150,0	
128	Investments		-350,0	-50,0	-50,0	0,0	0,0
129	Increase in working capital		-40,0	-60,0	-50,0	0,0	150,0
130	Free Cash Flow		-390,0	142,3	101,5	200,8	150,0
131							
132	**Determination of WACC year by year**						
133							
134	D/(D+E)		0,00	0,49	0,52	0,47	
135	E/(D+E)		1,00	0,51	0,48	0,53	
136							
137	WACC each period		11,0%	9,0%	8,9%	9,1%	
138							
139	WACC compounded	1,000	1,110	1,210	1,318	1,439	
140	Discount factor	1,000	0,901	0,826	0,759	0,695	0,695
141							
142	Discounted FCF to 31/12/2000		-351,4	117,5	77,0	139,5	104,3
143	PV of Cumulated discounted FCF	87,0					
144	PV of the Equity = NPV	**87,0**					
145							
146	Verification of the Equity value in each period:						
147	Total enterprise value at beginning of period		87,0	486,5	388,2	321,4	
148	Debt at beginning of period		0,0	240,0	200,0	150,0	
149	Equity at beginning of period		87,0	246,5	188,2	171,4	
150	*check with Equity from cash to equity at variable D/E*		*0,00000*	*0,00000*	*0,00000*	*0,00000*	

We would have attained the same result if, instead of introducing the notion of subsidy in the P&L, we had used the weighted average Kd year by year to calculate the WACC.

The valuation would have developed as set out below.

The construction of the Free Cash Flow in lines 200-205 would be the same as in the model under chapter 8.

The average Kd of line 213 is determined by dividing the total interest for the period by the debt of the period. Therefore, the average Kd ranges between 5% and 8% depending upon the relative weight of subsidized debt on total debt.

The WACC calculated in line 215 takes account of the average Kd and is slightly lower than the WACC calculated in chapter 8.

As was to be expected, here again the NPV is 87.0.

196	**WACC with average Kd**						
197							**Liquidation**
198	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
199							
200	EBIT		0,0	200,0	100,0	100,0	
201	Taxes on EBIT		0,0	-100,0	-50,0	-50,0	
202	Depreciation		0,0	150,0	150,0	150,0	
203	Investments		-350,0	-50,0	-50,0	0,0	0,0
204	Increase in working capital		-40,0	-60,0	-50,0	0,0	150,0
205	Free Cash Flow		-390,0	140,0	100,0	200,0	150,0
206							
207	**Determination of WACC year by year**						
208							
209	D/(D+E)		0,00	0,49	0,52	0,47	
210	E/(D+E)		1,00	0,51	0,48	0,53	
211							
212	Ke		11,0%	13,9%	14,2%	13,6%	
213	Average Kd			6,1%	6,5%	7,0%	
214							
215	WACC each period		11,0%	8,6%	8,6%	8,9%	
216							
217	WACC compounded	1,000	1,110	1,205	1,308	1,425	
218	Discount factor	1,000	0,901	0,830	0,764	0,702	0,702
219							
220	Discounted FCF on 31/12/2000		-351,4	116,2	76,4	140,4	105,3
221	PV of discounted FCF	87,0					
222	PV of the Equity = NPV	**87,0**					
223							

Another way to verify the correctness of the result consists in breaking it down into its component parts. For this reason, NPV values are shown to the second decimal point.

152	**Comparison with project with no subsidized debt**	
153		
154	**Equity value on 31/12/2000**	
155	NPV with no subsidised loan	**83,54**
156	NPV with subsidised loan	**86,96**
157	Difference	**3,42**
158		

In chapter 8 we have seen that the NPV without subsidized loan was 83.54.

As a result of the subsidized loan, the NPV has increased by 3.42, attaining 86.96.

The increase in the NPV can be ascribed to two factors. The lower interest paid compared to the market Kd, the so-called subsidy, and a lower risk for shareholders in that a less expensive loan results in a higher equity and hence a lower D/E and a less risky Ke. Let us now calculate these two effects.

The value of the subsidy is calculated in line 166 is equal to the subsidized debt for the period multiplied by the rate difference compared to the market Kd. The net effect on cash to equity is obtained when considering the subsidy after tax. Line 168.

160	**Effect of subsidised loan on cash to equity**					
161				**31/12/2002**	**31/12/2003**	**31/12/2004**
162	**Effect of lower interests**					
163	Difference between Kd at market rates and Kd subsidised			3,0%	3,0%	3,0%
164	Subsidised loan			150,0	100,0	50,0
165						
166	Interests saved on subsidized loan			4,50	3,00	1,50
167	Taxes on savings			-2,25	-1,50	-0,75
168	Net effect on cash to equity			2,25	1,50	0,75
169						
170						
171	Ke year by year		11,0%	13,9%	14,2%	13,6%
172	Discount factor	1,000	0,901	0,791	0,693	0,610
173	Discounted cash to equity		0,00	1,78	1,04	0,46
174						
175	Effect of saved interests on NPV		**3,28**			

Now take the Ke as determined in line 109 with the Cash To Equity methodology and discount the effect of the subsidy after tax. The resulting NPV is 3.28. This is the net value for shareholders arising from the lower interest rate of subsidized loan.

The effect of lower risk to shareholders can be verified by observing that the Ke determined in chapter 8 is slightly higher than the Ke determined with the subsidized loan.

Let us take the cash flows to the shareholders of the project without subsidized loan, line 184, and discount them using both the Ke of chapter 8, as shown in line 179, and the Ke obtained from the model with subsidized loan, line 180.

The difference between the two discounted values is 0.14. Line 191.

177	**Effect of lower risk**						
178							
179	Ke with loan at market conditions		11,000%	13,966%	14,222%	13,636%	
180	Ke with subsidized loan		11,000%	13,921%	14,188%	13,625%	
181							**Liquidation**
182				**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
183							
184	Cash to equity no subsidised loan		-150,0	90,4	42,0	44,0	150,0
185							
186	Discount factor with no subsidised loan	1,000	0,901	0,790	0,692	0,609	
187	Discount factor with subsidised loan	1,000	0,901	0,791	0,693	0,610	
188							
189	Discounted cash to equity		0,00	0,03	0,02	0,09	
190							
191	Effect of lower risk on NPV		**0,14**				

Indeed, the sum of 3.28 and 0.14 is 3.42.

As can be seen, there are various ways to verify the correctness of an NPV impacted by a subsidized loan.

Finally, let us try and apply the triangulation methodology and let us verify the NPV and the D/E representative of project ALFA with subsidized loan.

First off, it can be observed that it is not possible to apply the average Kd in that it changes from one year to the next. We must use the market Kd, which remains unaltered and applies throughout the project. As a result, for the WACC methodology, we will have to use the EBIT increased by the P&L subsidy implicit in the subsidized loan.

224	**Determination of the D/E representative of the project**							
225								
226	**ELEMENTS OF THE VALUATION**							
227		Kd	8%					
228		Rf	5%					
229		Rp	6%					
230		Taxes	50%					
231		F	1,41					
232		Beta Assets	1,00					
233		Beta Equity	1,41					
234		Ke	13,1%	representative of the project				
235		D/E	**0,82**	calculated with reiterations until the value of equity is the same				
236		D/(D+E)	0,45					
237		E/(D+E)	0,55					
238		WACC	9,0%	representative of the project				
239								**Liquidation**
240	**CASH TO EQUITY**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
241								
242	Cash to shareholders:			-150,0	92,7	43,5	44,8	150,0
243	Compounded Ke		1,000	1,131	1,278	1,445	1,634	
244	Discount factor		1,000	0,885	0,782	0,692	0,612	0,612
245	discounted cash to shareholders			-132,7	72,5	30,1	27,4	91,8
246								
247		PV of the Equity = NPV	**89,1**					
248								
249								**Liquidation**
250	**WACC**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
251								
252	Free Cash Flows			-390,0	142,3	101,5	200,8	150,0
253	Compounded WACC		1,000	1,090	1,187	1,294	1,410	
254	Discount factor		1,000	0,918	0,842	0,773	0,709	0,709
255	Discounted FCF			-357,9	119,8	78,4	142,4	106,4
256								
257		Enterprise value on 31/12/2000	89,1					
258		Debt on 31/12/2000	0,0					
259		PV of the Equity = NPV	**89,1**					
260								
261								
262		check of the reiteration	1,000	Reiterate D/E until the ratio between the equity value from Cash to Equity and				
263				from WACC have the same value				

The NPV is 89.1 (the difference compared to the NPV determined using the cash to equity methodology with a variable D/E being 2.5 %). The D/E representative of the project is now 0.82, which is, as should be, slightly lower than 0.84 as determined in chapter 8 without subsidized loan.

Here again, the triangulation methodology shows results consistent with - and not far from - exact values.

16. Enterprise Value Determination

Refer to "BRAVO no leverage" in the Spreadsheet.XLS file

Let us now apply the foregoing calculation methodologies to an enterprise case.

The date is 31.12.2000 and the company BRAVO is on sale. The owners have worked out a business plan that is shown to an investor. The three components of the business plan are, as is the rule, the P&L, the balance sheet and the cash flow statement and they rely upon the scenarios worked out by the management as to the business forecast.

36	**P&L**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
37						
38	Revenues		1.100,0	1.250,0	1.350,0	1.500,0
39	Costs		-900,0	-1.100,0	-1.100,0	-1.300,0
40	EBITDA		200,0	150,0	250,0	200,0
41	Depreciation		-40,0	-70,0	-70,0	-70,0
42	EBIT		160,0	80,0	180,0	130,0
43	Interests		-8,0	-16,8	-23,2	-29,6
44	EBT		152,0	63,2	156,8	100,4
45	Taxes		-76,0	-31,6	-78,4	-50,2
46	EAT		76,0	31,6	78,4	50,2
47						
48	**BALANCE SHEET**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
49						
50	Gross fixed assets	1.000,0	1.100,0	1.200,0	1.300,0	1.400,0
51	Cumulated depreciation	-450,0	-490,0	-560,0	-630,0	-700,0
52	Net Fixed Assets	550,0	610,0	640,0	670,0	700,0
53						
54	Working capital	150,0	200,0	250,0	300,0	350,0
55	TOTAL ASSETS	700,0	810,0	890,0	970,0	1.050,0
56						
57	Debt	100,0	210,0	290,0	370,0	450,0
58	Share capital	600,0	600,0	600,0	600,0	600,0
59	TOTAL LIABILITIES	700,0	810,0	890,0	970,0	1.050,0
60	*check*	*0,000*	*0,000*	*0,000*	*0,000*	*0,000*
61						
62	**CASH FLOW STATEMENT**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
63						
64	Sources of funds					
65	EAT		76,0	31,6	78,4	50,2
66	Depreciation		40,0	70,0	70,0	70,0
67	Equity contribution		0,0	0,0	0,0	0,0
68	**Total sources**		116,0	101,6	148,4	120,2
69	Uses of funds					
70	Investments		100,0	100,0	100,0	100,0
71	Increase in working capital		50,0	50,0	50,0	50,0
72	Dividends and capital distribution		76,0	31,6	78,4	50,2
73	**Total uses**		226,0	181,6	228,4	200,2
74						
75	**Sources minus uses of funds**		-110,0	-80,0	-80,0	-80,0
76						
77	Net cash position beginning of period		-100,0	-210,0	-290,0	-370,0
78	Sources minus uses of funds		-110,0	-80,0	-80,0	-80,0
79	Net cash ending of period	-100,0	-210,0	-290,0	-370,0	-450,0

The investor, analysing the plan, estimates that the enterprise can be re-sold at the end of 2004 for a total enterprise value of 1200.

Though this chapter does not intend to discuss how to determine the long-term value of an enterprise, this principle is briefly illustrated here.

When working out a business plan, a fairly crucial point for all analysts is to assign a value to the enterprise for the years following after the last year of the business plan. This value is referred to as terminal value. This topic is of the essence. The analyst develops a business model that projects the enterprise behaviour over a number of years during which it is assumed that forecasts can be made, if nothing else; then, it is necessary to put forward hypotheses as to the value of the enterprise at the end of the business plan.

Normally, these are the scenarios used to determine the terminal value:

16.1 Transfer

This is commonly used by financial institutions and investment funds. In practice, the analyst assumes the transfer of the company for a given price at the end of the business plan. In this case, the reasoning to determine the terminal value hinges upon the market conditions at the time of transfer. It is fairly common to find terminal values based upon profitability multiples like "n" times EBITDA or a given P/E (Price/Earning after tax), or upon book values increased by goodwill.

16.2 Perpetuity

This is used by those industrial investors that do not necessarily envisage the transfer of the company at the end of the business plan. In this case, the analyst assumes that the company continues to generate a given cash flow indefinitely and valorises it by discounting it as if it were a perpetuity. With the perpetuity scenario it is also possible to factor in a certain (perpetual) growth of the enterprise. The use of perpetuity is delicate. On the one hand it raises fundamental, almost philosophical, issues. For instance, on the grounds of what can it be stated that an enterprise will exist and grow infinitely? On the other hand, it can lead to very large, and hence hardly credible, terminal values.

16.3 Liquidation

This is the most conservative and penalising scenario for the NPV of the investment. In this case, it is assumed that the shareholders will only receive the proceeds from liquidation at the end of the business plan. The analysis of project ALFA in the previous chapters was based, for the sake of simplicity, on this scenario. Still, the valuation of an enterprise assuming liquidation of the enterprise at the end of the business plan is a fairly rare occurrence.

There is no single best method, amongst the ones illustrated above. It is up to the analyst to use his/her commonsense and caution in choosing the scenario underlying the valuation. In our case the investor estimated that it could re-sell the company at 6x the EBITDA of the last year of the business plan, i.e. 6 x 200 = 1200 Total Enterprise Value.

The variables for calculating Ke and WACC are as follows:

1	**INPUT ZONE**		
2			
3	Kd	input	8,0%
4	Rp	input	6,0%
5	Rf	input	5,0%
6	Taxes	input	50,0%
7	Beta assets	input	1,000

Let us now see the equity value of BRAVO as at 31.12.2000 (this being the date of transfer). Obviously, we are referring to the value for the transferor, i.e. the shareholders.

Let us verify all of the methods available.

The Cash To Equity methodology with variable D/E shows in line 104 an equity NPV of 657.1.

81	**Cash to Equity methodology with variable D/E year by year**						
82							**Exit value**
83	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
84							
85	Debt at liquidation						450,0
86	Equity at liquidation						750,0
87	Total enterprise value at liquidation (D+E)						1.200,0
88	Debt at beginning of period		100,0	210,0	290,0	370,0	-450,0
89	Cash to shareholders:						
90	(dividends + capital distributions - equity contribution)		76,0	31,6	78,4	50,2	750,0
91							
92							
93	**Determination of D/E, F and Ke year by year**		**2001**	**2002**	**2003**	**2004**	
94							
95	Kd		8%	8%	8%	8%	
96	Rp		6%	6%	6%	6%	
97	Rf		5%	5%	5%	5%	
98	Taxes		50%	50%	50%	50%	
99	F		1,08	1,16	1,21	1,26	
100	Beta assets		1,00	1,00	1,00	1,00	
101	Beta equity		1,08	1,16	1,21	1,26	
102	Ke		11,5%	12,0%	12,2%	12,6%	
103							
104	PV of Equity at the beginning of each period		**657,1**	656,3	703,2	710,9	
105							
106	D/E of each period		0,15	0,32	0,41	0,52	
107							
108	Note: the value of the equity at the beginning of one period is calculated by discounting one						
109	year the cash flow to shareholders at the end of the period together with the value of the						
110	equity calculated for the following period with the same methodology						
111							
112	Total enterprise value (D+E) at beginning of the period		757,1	866,3	993,2	1.080,9	

The Ke is calculated on every period using the formula under chapter 8 included in line 102.

The equity value being higher than the book value, the transferors will realize a capital gain. The overestimation of equity at the time of a transaction is referred to as goodwill.

We can now apply the WACC methodology with variable D/E.

114	**WACC methodology with variable D/E year by year**						
115							**Exit value**
116	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
117							
118	EBIT		160,0	80,0	180,0	130,0	
119	Taxes on EBIT		-80,0	-40,0	-90,0	-65,0	
120	Depreciation		40,0	70,0	70,0	70,0	
121	Investments		-100,0	-100,0	-100,0	-100,0	
122	Increase in working capital		-50,0	-50,0	-50,0	-50,0	
123	Free Cash Flow		-30,0	-40,0	10,0	-15,0	1.200,0
124							
125	**Determination of WACC year by year**						
126	D/(D+E)		0,13	0,24	0,29	0,34	
127	E/(D+E)		0,87	0,76	0,71	0,66	
128							
129	WACC of each period		10,5%	10,0%	9,8%	9,6%	
130	WACC compounded	1,000	1,105	1,216	1,335	1,464	
131	Discount factor	1,000	0,905	0,823	0,749	0,683	0,683
132							
133	Discounted FCF to 31/12/2000		-27,2	-32,9	7,5	-10,2	819,9
134	PV of Cumulated discounted FCF	757,1					
135	Debt on 31/12/2000	100,0					
136	PV of the Equity	**657,1**					
137							
138	Verification of the Equity value in each period:						
139	Total enterprise value at beginning of period		757,1	866,3	993,2	1.080,9	
140	Debt at beginning of period		100,0	210,0	290,0	370,0	
141	Equity at beginning of period (TEP -Debt)		657,1	656,3	703,2	710,9	
142	*check with Equity from cash to equity at variable D/E*		*0,00000*	*0,00000*	*0,00000*	*0,00000*	

Here again, in line 136 the equity NPV as at 31.12.2000 is 657.1.

As with the valuation of a project, also with an enterprise the two methodologies (cash to equity and WACC) lead to exactly the same result.

Let us now determine the D/E representative of the enterprise using the triangulation methodology. The relevant table is shown on the next following page.

Let us set the target function so as to find in line 156 a D/E that leads to the same NPVs in lines 168 and 179.

The D/E representative of the enterprise is 0.34, which corresponds to a Ke of 12.0% and a WACC of 10.0%

The NPV calculated using such D/E value that remains constant throughout the valuation period is 657.0, which only marginally differs from the result obtained with the two previous methodologies.

145	**Determination of the representative D/E**							
146								
147	**ELEMENTS OF THE VALUATION**							
148		Kd	8%					
149		Rf	6%					
150		Rp	5%					
151		Taxes	50%					
152		F	1,17					
153		Beta Assets	1,00					
154		Beta Equity	1,17					
155		Ke	12,0%	representative of all the periods				
156		D/E	**0,34**	calculated with reiterations				
157		D/(D+E)	0,25					
158		E/(D+E)	0,75					
159		WACC	10,0%	representative of all the periods				
160								
161	**CASH TO EQUITY**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
162								
163	Cash to shareholders:			76,0	31,6	78,4	50,2	750,0
164	Compounded Ke		1,000	1,120	1,255	1,406	1,575	
165	Discount factor		1,000	0,893	0,797	0,711	0,635	0,635
166	discounted cash to shareholders			67,8	25,2	55,8	31,9	476,3
167								
168		PV of the Equity on 31/12/2000	**657,0**					
169								
170	**WACC**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
171								
172	Free Cash Flows			-30,0	-40,0	10,0	-15,0	1.200,0
173	Compounded WACC		1,000	1,100	1,210	1,330	1,463	1,609
174	Discount factor		1,000	0,909	0,827	0,752	0,683	0,683
175	Discounted FCF			-27,3	-33,1	7,5	-10,3	820,1
176								
177		Total Enterprise Value on 31/12/2000	757,0					
178		Debt on 31/12/2000	-100,0					
179		PV of the Equity on 31/12/2000	**657,0**					
180								
181		check of the reiteration	1,000	Reiterate D/E until the ratio between the equity value from Cash to Equity and				
182				from WACC have the same value				

With the WACC determined as above, let us calculate the EVA of the enterprise on the net invested capital:

241	**Determination of EVA on NIC using WACC =**	**10,0%**	from the triangulation method					
242								
243								**exit value**
244			**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
245	NIC beginning of the period			700,0	810,0	890,0	970,0	
246	expected NOPAT = NIC x WACC			69,9	80,9	88,9	96,9	
247	achieved NOPAT = EBIT*(1-T)			80,0	40,0	90,0	65,0	
248		EVA		10,1	-40,9	1,1	-31,9	150,0
249								
250	Discount factor based on WACC		1	0,909	0,827	0,752	0,683	0,683
251	Discounted EVA			9,2	-33,8	0,9	-21,8	102,5
252								
253		Present Value of EVA	57,0					
254		NIC on 31/12/2000	700,0					
255		Debt on 31/12/2000	-100,0					
256		PV of the Equity on 31/12/2000	**657,0**					

For the sake of verification, let us determine the EVA on the gross invested capital.

258	**Determination of EVA on GIC using WACC =**	**10,0%**	from the triangulation method					
259								
260								**exit value**
261			**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
262	GIC beginning of period			1.150,0	1.300,0	1.450,0	1.600,0	
263	expected NOPAT = GIC x WACC			114,8	129,8	144,8	159,8	
264	achieved NOPAT + Depreciation			120,0	110,0	160,0	135,0	
265	EVA			5,2	-19,8	15,2	-24,8	-550,0
266								
267	Discount factor based on WACC		1,000	0,909	0,827	0,752	0,683	0,683
268	Discounted EVA			4,7	-16,4	11,4	-16,9	-375,9
269								
270	Present Value of EVA		-393,0					
271	GIC on 31/12/2000		1.150,0					
272	Debt on 31/12/2000		-100,0					
273	PV of the Equity on 31/12/2000		**657,0**					

As can be seen, the calculations lead at all times to the same value, i.e. 657.0, in line 256 as well as in line 273.

Finally, let us verify that by using book values of D/E the results obtained are mutually inconsistent. Indeed, using the book-value D/E of each period, the NPV as determined with the Cash To Equity methodology is 654, line 209, and 660 with WACC, line 220.

184	**Determination of Ke and WACC using book values**		**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	
185							
186	Debt at beginning of the period		100,0	210,0	290,0	370,0	
187	Book value of equity at beginning of period		600,0	600,0	600,0	600,0	
188	Total enterprise value at beginning of period		700,0	810,0	890,0	970,0	
189							
190	Kd		8%	8%	8%	8%	
191	Rf		6%	6%	6%	6%	
192	Rp		5%	5%	5%	5%	
193	Taxes		50%	50%	50%	50%	
194	F		1,08	1,18	1,24	1,31	
195	Beta Assets		1,00	1,00	1,00	1,00	
196	Beta Equity		1,08	1,18	1,24	1,31	
197	Ke at book value year by year		11,5%	12,1%	12,5%	12,9%	
198	D/E book value		0,167	0,350	0,483	0,617	
199	D/(D+E) book value		14,3%	25,9%	32,6%	38,1%	
200	E/(D+E) book value		85,7%	74,1%	67,4%	61,9%	
201	WACC at book value year by year		10,4%	10,0%	9,7%	9,5%	
202							**exit value**
203	**CASH TO EQUITY**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
204	**Variable Ke year by year**						
205	Cash to shareholders:		76,0	31,6	78,4	50,2	750,0
206	Compounded Ke	1,000	1,115	1,249	1,405	1,585	
207	Discount factor	1,000	0,897	0,800	0,712	0,631	0,631
208	Discounted cash to shareholders		68,2	25,3	55,8	31,7	473,1
209	PV of the Equity on 31/12/2000	**654,0**					
210							**exit value**
211	**WACC**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
212	**Variable WACC year by year**						
213	Free Cash Flow		-30,0	-40,0	10,0	-15,0	1.200,0
214	Compounded WACC	1,000	1,104	1,214	1,332	1,458	1,458
215	Discount factor	1,000	0,906	0,824	0,751	0,686	0,686
216	Discounted FCF		-27,2	-32,9	7,5	-10,3	822,9
217							
218	TEP on 31/12/2000	760,0					
219	Debt on 31/12/2000	-100,0					
220	PV of the Equity on 31/12/2000	**660,0**					

If, instead, we use the D/E of 0.167 of year 1, in line 198 under the column as at 31.12.2001, the NPV is 667.9 using the cash to equity methodology, line 228, and 644.3 using WACC, line 239.

221							exit value
222	**CASH TO EQUITY**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
223	**Constant Ke equal to 1st year**						
224	Cash to shareholders:		76,0	31,6	78,4	50,2	750,0
225	Compounded Ke	1,000	1,115	1,243	1,386	1,546	
226	Discount factor	1,000	0,897	0,804	0,721	0,647	0,647
227	Discounted cash to shareholders		68,2	25,4	56,6	32,5	485,2
228	PV of the Equity on 31/12/2000	**667,9**					
229							exit value
230	**WACC**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
231	**Constant WACC equal to 1st year**						
232	Free Cash Flow		-30,0	-40,0	10,0	-15,0	1.200,0
233	Compounded WACC	1,000	1,104	1,219	1,347	1,487	1,642
234	Discount factor	1,000	0,906	0,820	0,743	0,672	0,672
235	Discounted FCF		-27,2	-32,8	7,4	-10,1	807,0
236							
237	TEP on 31/12/2000	744,3					
238	Debt on 31/12/2000	-100,0					
239	PV of the Equity on 31/12/2000	**644,3**					

In summary, these are all the NPVs calculated.

26	**OUTPUT ZONE**				
27					
28	**Equity value on 31/12/2000 using D/E at proxy market value**	**NPV**	**Equity value on 31/12/2000 using book value D/E**	**NPV**
29	Cash to equity with variable D/E	**657,1**	Cash to equity with variable D/E	**654,0**
30	WACC with variable D/E	**657,1**	WACC with variable D/E	**660,0**
31	Triangulation	**657,0**	Cash to equity with constant Ke equal to 1st year	**667,9**
32	EVA on Net Invested Capital	**657,0**	WACC with constant WACC equal to 1st year	**644,3**
33	EVA on Gross Invested Capital	**657,0**		

Here again, the use of D/E at proxy market value is more precise than - and consistent with - the use of book-value D/E.

Eventually, the investor decides to purchase 100% of the BRAVO shares for a value of 657, i.e. the discounted cash flow value that the transferor shareholders would have received according to the business plan if they had decided not to transfer the enterprise.

In practice, the purchaser has invested a principal of 657 in an enterprise that promises a ROI of 12.0% while risk-free return would attain a mere 5%. The transaction, per se, does not entail any further value creation for the investor. Indeed, the NPV of the acquisition transaction is zero. 657 -657 = 0

17. Acquisition with Leverage

Refer to "BRAVO with leverage" in Spreadsheet.XLS file

At the time of acquisition, the investor has an idea. It believes that BRAVO can, in fact, operate with an indebtedness exceeding the 0.34 D/E as determined with the triangulation methodology in the previous chapter.

To do so, the investor establishes NewCo, the vehicle for the acquisition.
NewCo is financed with equity (200) and debt (457) and uses cash (657) to finalise the acquisition of BRAVO.

Balance Sheet of NewCo prior to the acquisition:

Assets	**Liabilities**
Cash = 657	**Debt = 457**
	Equity = 200

***Balance Sheet of NewCo after the acquisition of BRAVO*:**

Assets	**Liabilities**
100% BRAVO shares	**Debt = 457**
	Equity = 200

Then, BRAVO is merged into NewCo and NewCo changes the name to BRAVO. The financial structure of BRAVO resulting from the merger has much more debt than the previous one.

Suppose that all these transactions occur as at 31.12.2000 at the time of acquisition.

Also suppose that the purchaser has agreed with banks that the Kd of post-merger debt increases by 1% attaining 9%. Furthermore, the purchaser undertakes a commitment with banks whereby it will not distribute any dividend in the years to come until the time of exit (this being the covenant of the much more heavily indebted financial structure).

As a result, starting from 1.1.2001 the business plan of BRAVO will be impacted by the new starting balance sheet and by the covenant on dividends.

Lines 47, 49 and 50 of P&L change as a result of higher interest.

40	**P&L after merger with NewCo**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
41						
42	Revenues		1.100,0	1.250,0	1.350,0	1.500,0
43	Costs		-900,0	-1.100,0	-1.100,0	-1.300,0
44	EBITDA		200,0	150,0	250,0	200,0
45	Depreciation		-40,0	-70,0	-70,0	-70,0
46	EBIT		160,0	80,0	180,0	130,0
47	Interests		-50,1	-55,1	-61,2	-63,0
48	EBT		109,9	24,9	118,8	67,0
49	Taxes		-54,9	-12,5	-59,4	-33,5
50	EAT		54,9	12,5	59,4	33,5
51						

The balance sheet changes as well, due to the addition, in line 57, of the goodwill paid to purchase BRAVO.

Likewise, debt on line 62 and equity on line 63 have changed.

For the sake of simplicity, we have assumed that the amortisation of goodwill is not fiscally deductible; as a result, it has not been amortised and has been left constant.

52	**BALANCE SHEET after merger with NewCo**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
53						
54	Gross fixed assets	1.000,0	1.100,0	1.200,0	1.300,0	1.400,0
55	Cumulated depreciation	-450,0	-490,0	-560,0	-630,0	-700,0
56	Net Fixed Assets	550,0	610,0	640,0	670,0	700,0
57	Goodwill	57,0	57,0	57,0	57,0	57,0
58	Working capital	150,0	200,0	250,0	300,0	350,0
59						
60	TOTAL ASSETS	757,0	867,0	947,0	1.027,0	1.107,0
61						
62	Debt	557,0	612,1	679,6	700,2	746,7
63	Share capital and reserves	200,0	254,9	267,4	326,8	360,3
64	TOTAL LIABILITIES	757,0	867,0	947,0	1.027,0	1.107,0
65	*check*	*0,000*	*0,000*	*0,000*	*0,000*	*0,000*

The cash flow statement reflects all of the transactions described until now.

The purchase transactions by NewCo and the merger transactions with BRAVO are highlighted in column 31.12.2000.

In line 72, the equity of 200 is the one paid by purchaser. Instead, in line 77 the distribution of cash to shareholders is the cash to transferor shareholders.

67	**CASH FLOW STATEMENT after merger with NewCo**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**
68						
69	Sources of funds					
70	EAT		54,9	12,5	59,4	33,5
71	Depreciation		40,0	70,0	70,0	70,0
72	Equity contribution	200,0	0,0	0,0	0,0	0,0
73	**Total sources**	200,0	94,9	82,5	129,4	103,5
74	Uses of funds					
75	Investments		100,0	100,0	100,0	100,0
76	Increase in working capital		50,0	50,0	50,0	50,0
77	Dividends and capital distribution	657,0	0,0	0,0	0,0	0,0
78	**Total uses**	657,0	150,0	150,0	150,0	150,0
79						
80	**Sources minus uses of funds**	-457,0	-55,1	-67,5	-20,6	-46,5
81						
82	Net cash position beginning of period	-100,0	-557,0	-612,1	-679,6	-700,2
83	Sources minus uses of funds	-457,0	-55,1	-67,5	-20,6	-46,5
84	Net cash ending of period	-557,0	-612,1	-679,6	-700,2	-746,7

The table with the fundamentals of risk has changed as a result of the higher Kd.

Therefore, we can determine Ke and WACC after the merger of BRAVO with NewCo.

1	**INPUT ZONE**			
2				
3	Kd	input	9,0%	
4	Rp	input	6,0%	
5	Rf	input	5,0%	
6	Taxes	input	50,0%	
7	Beta assets	input	1,000	
8	Terminal value	input	1200	estimated total enterprise value

All of the other values of the business plan remain unaltered in that they are not impacted by debt. The same holds true for the terminal value, which is equal to 6x the EBITDA of the last year.

Let us now verify if this financial strategy creates value for the investor that has conceived it.

First, let us calculate the equity value using the Cash To Equity methodology.

86	**Cash to Equity methodology with variable D/E year by year**						
87							**Exit value**
88	**ELEMENTS OF THE VALUATION**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
89							
90	Debt at liquidation						746,7
91	Equity at liquidation						453,3
92	Total enterprise value at liquidation (D+E)						1.200,0
93	Debt at beginning of period		557,0	612,1	679,6	700,2	746,7
94	Cash to shareholders:						
95	(dividends + capital distributions - equity contribution)	-200,0	0,0	0,0	0,0	0,0	453,3
96							
97							
98	**Determination of D/E, F and Ke year by year**		**2001**	**2002**	**2003**	**2004**	
99							
100	Kd		9%	9%	9%	9%	
101	Rp		6%	6%	6%	6%	
102	Rf		5%	5%	5%	5%	
103	Taxes		50%	50%	50%	50%	
104	F		2,16	2,08	2,02	1,90	
105	Beta assets		1,00	1,00	1,00	1,00	
106	Beta equity		2,16	2,08	2,02	1,90	
107	Ke		18,0%	17,5%	17,1%	16,4%	
108							
109	PV of Equity at the beginning of each period	**39,9**	239,9	283,0	332,5	389,5	
110							
111	D/E of each period		2,32	2,16	2,04	1,80	
112							
113	Note: the value of the equity at the beginning of one period is calculated by discounting one						
114	year the cash flow to shareholders at the end of the period together with the value of the						
115	equity calculated for the following period with the same methodology						
116							
117	Total enterprise value (D+E) at beginning of the period		796,9	895,1	1.012,1	1.089,6	

Net of the cash payment of 200 for the acquisition of BRAVO, the NPV accrued to the investor is 39.9. Line 109.

This is entirely due to the new financial structure and to the tax shield arisen from interest, duly discounted with Ke.

The same result is obtained using the WACC methodology.

119	**WACC methodology with variable D/E year by year**							
120								**Exit value**
121	**ELEMENTS OF THE VALUATION**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
122								
123	EBIT			160,0	80,0	180,0	130,0	
124	Taxes on EBIT			-80,0	-40,0	-90,0	-65,0	
125	Depreciation			40,0	70,0	70,0	70,0	
126	Investments			-100,0	-100,0	-100,0	-100,0	
127	Increase in working capital			-50,0	-50,0	-50,0	-50,0	
128		Free Cash Flow	-200,0	-30,0	-40,0	10,0	-15,0	1.200,0
129								
130	**Determination of WACC year by year**							
131	D/(D+E)			0,70	0,68	0,67	0,64	
132	E/(D+E)			0,30	0,32	0,33	0,36	
133								
134	WACC each period			8,6%	8,6%	8,6%	8,8%	
135	WACC compounded		1,000	1,086	1,179	1,281	1,393	
136	Discount factor		1,000	0,921	0,848	0,781	0,718	0,718
137								
138	Discounted FCF to 31/12/2000		-200,0	-27,6	-33,9	7,8	-10,8	861,4
139	PV of Cumulated discounted FCF		596,9					
140	Debt on 31/12/2000		557,0					
141	PV of the Equity		**39,9**					
142								
143	Verification of the Equity value in each period:							
144	Total enterprise value at beginning of period			796,9	895,1	1.012,1	1.089,6	
145	Debt at beginning of period			557,0	612,1	679,6	700,2	
146	Equity at beginning of period			239,9	283,0	332,5	389,5	
147	*check with Equity from cash to equity at variable D/E*			*0,00000*	*0,00000*	*0,00000*	*0,00000*	

Please note that the WACC is almost 1.5% lower than that used for the valuation of BRAVO with the originally lower level of debt.

Finally, using the triangulation methodology, we can verify that the D/E representative of BRAVO after financial restructuring is 2.08, much higher compared to 0.34 in the previous chapter.

The NPV of 40.0 calculated using triangulation only marginally differs from that of the two previous methodologies.

150	**Determination of the D/E representative of the company**							
151								
152	**ELEMENTS OF THE VALUATION**							
153		Kd	9%					
154		Rf	6%					
155		Rp	5%					
156		Taxes	50%					
157		F	2,04					
158		Beta Assets	1,00					
159		Beta Equity	2,04					
160		Ke	17,2%	representative of the project				
161		D/E	**2,08**	calculated with reiterations until the value of equity is the same				
162		D/(D+E)	0,67					
163		E/(D+E)	0,33					
164		WACC	8,6%	representative of the project				
165								
166	**CASH TO EQUITY**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
167								
168	Cash to shareholders:		-200,0	0,0	0,0	0,0	0,0	453,3
169	Compounded Ke		1,000	1,172	1,374	1,611	1,889	
170	Discount factor		1,000	0,853	0,728	0,621	0,530	0,530
171	discounted cash to shareholders		-200,0	0,0	0,0	0,0	0,0	240,0
172								
173		PV of the Equity on 31/12/2000	**40,0**					
174								
175	**WACC**		**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
176								
177	Free Cash Flows		-200,0	-30,0	-40,0	10,0	-15,0	1200,0
178	Compounded WACC		1,000	1,086	1,180	1,282	1,393	1,513
179	Discount factor		1,000	0,920	0,847	0,780	0,718	0,718
180	Discounted FCF		-200,0	-27,6	-33,9	7,8	-10,8	861,5
181								
182		TEP value on 31/12/2000	597,0					
183		Debt on 31/12/2000	-557,0					
184		Equity value on 31/12/2000	**40,0**					
185								
186		check of the reiteration	1,000	Reiterate D/E until the ratio between the equity value from Cash to Equity and				
187				from WACC have the same value				

For the sake of completeness, let us also determine the same value using EVA on the net invested capital and on the gross invested capital.

246	**Determination of EVA on NIC using WACC =**	**8,6%**	from the triangulation method					
247								
248			**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
249	NIC beginning of the period			757,0	867,0	947,0	1.027,0	
250	expected NOPAT = NIC x WACC			65,4	74,9	81,8	88,7	
251	achieved NOPAT = EBIT*(1-T)			80,0	40,0	90,0	65,0	
252	EVA			14,6	-34,9	8,2	-23,7	93,0
253								
254	Discount factor = WACC compounded		1,000	0,920	0,847	0,780	0,718	0,718
255	Discounted EVA			13,4	-29,6	6,4	-17,0	66,8
256								
257	Present Value of EVA		40,0					
258	NIC on 31/12/2000		757,0					
259	Debt on 31/12/2000		-557,0					
260	Equity contribution on 31/12/2000		-200,0					
261	NPV of the leveraging		**40,0**					
262								
263	**Determination of EVA on GIC using WACC =**	**8,6%**	from the triangulation method					
264								
265			**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
266	GIC beginning of period			1.207,0	1.357,0	1.507,0	1.657,0	
267	expected NOPAT = GIC x WACC			104,3	117,2	130,2	143,1	
268	achieved NOPAT + Depreciation			120,0	110,0	160,0	135,0	
269	EVA			15,7	-7,2	29,8	-8,1	-607,0
270								
271	Discount factor = WACC compounded		1,000	0,920	0,847	0,780	0,718	0,718
272	Discounted EVA			14,5	-6,1	23,3	-5,8	-435,8
273								
274	Present Value of EVA		-410,0					
275	GIC on 31/12/2000		1.207,0					
276	Debt on 31/12/2000		-557,0					
277	Equity contribution on 31/12/2000		-200,0					
278	NPV of the leveraging		**40,0**					

Finally, let us verify once again that book-value D/E leads to mutually inconsistent and different values.

189	**Determination of Ke and WACC using book values**						
190			**2001**	**2002**	**2003**	**2004**	
191	Debt at beginning of the period		557,0	612,1	679,6	700,2	
192	Book value of equity at beginning of period		200,0	254,9	267,4	326,8	
193	Total enterprise value at beginning of period		757,0	867,0	947,0	1.027,0	
194							
195	Kd		9%	9%	9%	9%	
196	Rf		6%	6%	6%	6%	
197	Rp		5%	5%	5%	5%	
198	Taxes		50%	50%	50%	50%	
199	F		2,39	2,20	2,27	2,07	
200	Beta Assets		1,00	1,00	1,00	1,00	
201	Beta Equity		2,39	2,20	2,27	2,07	
202	Ke at book value year by year		19,4%	18,2%	18,6%	17,4%	
203	D/E book value		2,785	2,401	2,542	2,143	
204	D/(D+E) book value		73,6%	70,6%	71,8%	68,2%	
205	E/(D+E) book value		26,4%	29,4%	28,2%	31,8%	
206	WACC at book value year by year		8,4%	8,5%	8,5%	8,6%	
207							**exit value**
208	**CASH TO EQUITY**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
209	**Variable Ke year by year**						
210	Cash to shareholders:	-200,0	0,0	0,0	0,0	0,0	453,3
211	Compounded Ke	1,000	1,194	1,411	1,674	1,965	
212	Discount factor	1,000	0,838	0,709	0,598	0,509	0,509
213	Discounted cash to shareholders	-200,0	0,0	0,0	0,0	0,0	230,7
214	PV of the Equity on 31/12/2000	**30,7**					
215							**exit value**
216	**WACC**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
217	**Variable WACC year by year**						
218	Free Cash Flow	-200,0	-30,0	-40,0	10,0	-15,0	1.200,0
219	Compounded WACC	1,000	1,084	1,177	1,277	1,387	1,387
220	Discount factor	1,000	0,922	0,850	0,783	0,721	0,721
221	Discounted FCF	-200,0	-27,7	-34,0	7,8	-10,8	865,4
222							
223	TEP on 31/12/2000	600,8					
224	Debt on 31/12/2000	-557,0					
225	PV of the Equity on 31/12/2000	**43,8**					
226							**exit value**
227	**CASH TO EQUITY**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
228	**Constant Ke equal to 1st year**						
229	Cash to shareholders:	-200,0	0,0	0,0	0,0	0,0	453,3
230	Compounded Ke	1,000	1,194	1,425	1,700	2,029	
231	Discount factor	1,000	0,838	0,702	0,588	0,493	0,493
232	Discounted cash to shareholders	-200,0	0,0	0,0	0,0	0,0	223,4
233	PV of the Equity on 31/12/2000	**23,4**					
234							**exit value**
235	**WACC**	**31/12/2000**	**31/12/2001**	**31/12/2002**	**31/12/2003**	**31/12/2004**	**1/1/2005**
236	**Constant WACC equal to 1st year**						
237	Free Cash Flow	-200,0	-30,0	-40,0	10,0	-15,0	1.200,0
238	Compounded WACC	1,000	1,084	1,176	1,275	1,382	1,498
239	Discount factor	1,000	0,922	0,851	0,785	0,724	0,724
240	Discounted FCF	-200,0	-27,7	-34,0	7,8	-10,9	868,3
241							
242	TEP on 31/12/2000	603,6					
243	Debt on 31/12/2000	-557,0					
244	PV of the Equity on 31/12/2000	**46,6**					

Set out below is the synoptic table with all of the results. Please note that in this case, with low NPVs, the discrepancy of results in terms of book-value D/E is huge. NPV ranges from 23.4 to 46.6.

Instead, with the methodologies using D/E at proxy market value, the NPVs are much more repetitive and reliable.

30	**OUTPUT ZONE**				
31					
32	**Equity value on 31/12/2000 using D/E at proxy market value**	**NPV**		**Equity value on 31/12/2000 using book value D/E**	**NPV**
33	Cash to equity with variable D/E	**39,9**		Cash to equity with variable D/E	**30,7**
34	WACC with variable D/E	**39,9**		WACC with variable D/E	**43,8**
35	Triangulation	**40,0**		Cash to equity with constant Ke equal to 1st year	**23,4**
36	EVA on Net Invested Capital	**40,0**		WACC with constant WACC equal to 1st year	**46,6**
37	EVA on Gross Invested Capital	**40,0**			

Therefore, it can be maintained that the financial restructuring conceived by purchaser entails a higher equity value for the investor and hence creates value.

In practice, the shareholder-investor invests 200 of own equity and is financed by banks for the remaining part of the investment. It is subject to a much riskier Ke, which, in our case, is about 17.2% against 12% originally in BRAVO with little leverage as described in the previous chapter. But if everything works it gains higher returns and has an additional NPV of 40.

Please note that, at anyhow, the exit value for the previous shareholders is 657; therefore, the NPV of 40 arising from the financial structure is a real creation of additional enterprise value prior to restructuring.

Obviously, this is a bet, somewhat as described in chapter 1. It is necessary that also the banks that will supply the loan believe in it and it is further necessary that the business plan rests on a sound foundation and that the development over time is as expected.

Most private equity funds, Leveraged Buy Out transactions (LBOs) and Management Buy Out transactions (MBOs) rely on these principles and, at times, considerable value is created.

18. Conclusions and Suggestions

Hopefully this manual has shown the importance of some notions of finance and discounting techniques to be taken account of when valuating business models:

1. **Business models should be concise but should exhaustively represent the investment: P&L, balance sheet, cash flow statement.**
2. **Account should be taken of risk, which should be factored into discount rates.**
3. **Do not use book-value D/E.**
4. **Use the triangulation methodology to determine the representative D/E and the resulting NPV.**
5. **If need be, verify the exactness of the NPV using the WACC methodology and the Cash To Equity methodology with variable D/E.**

Wishing all the best for their work to those who will apply these principles, I would like to remind the readers that a manual and, in general, calculation tools are totally useless unless they are applied with commonsense and consistency.

